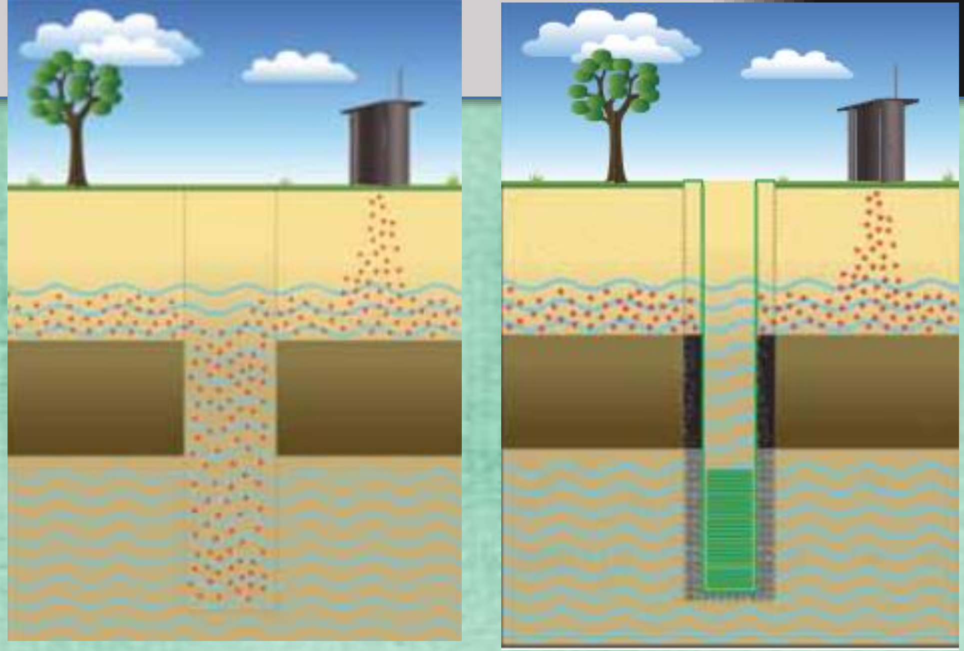
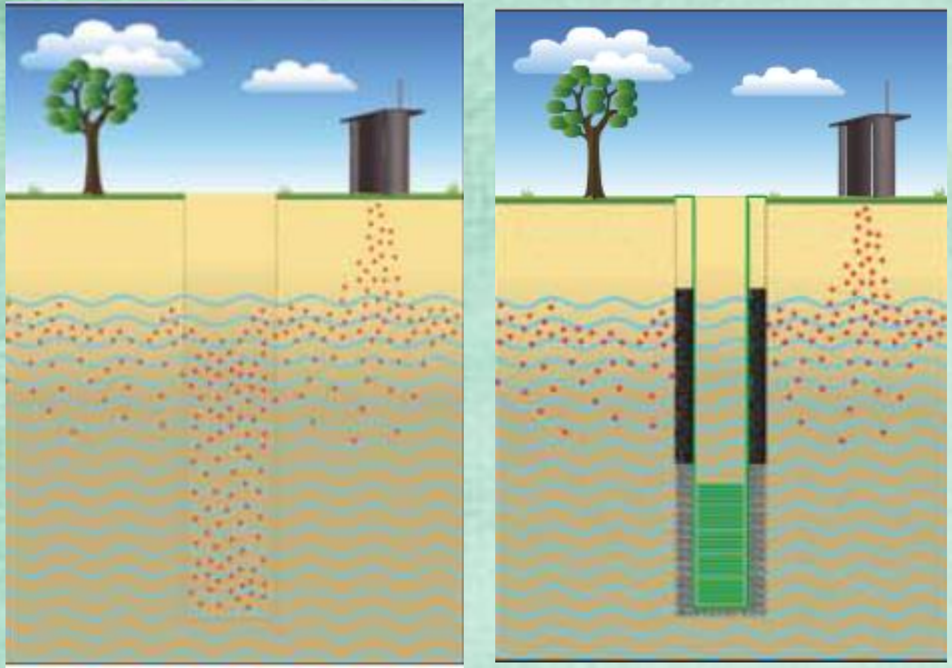
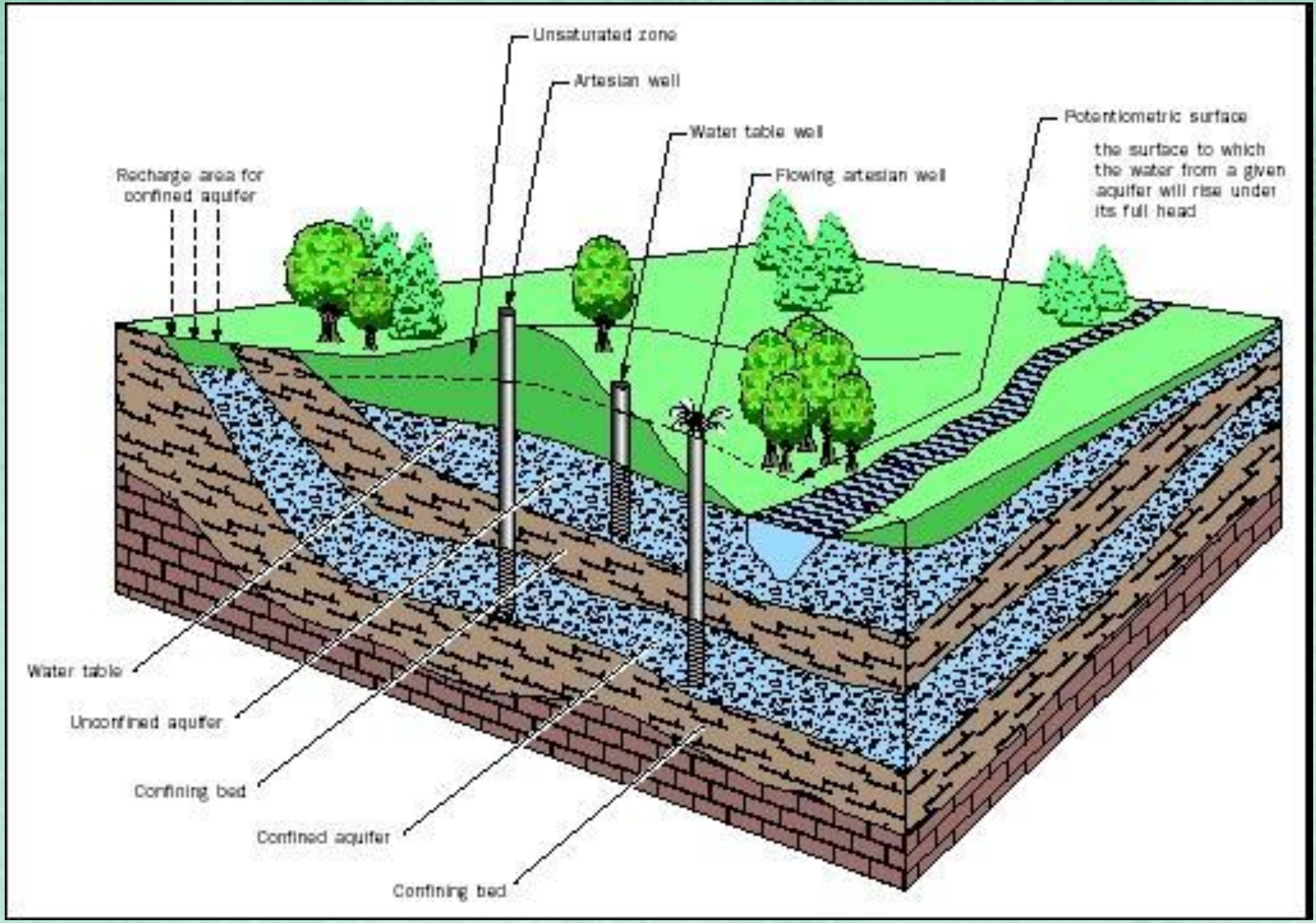


Van der Wal (2010)

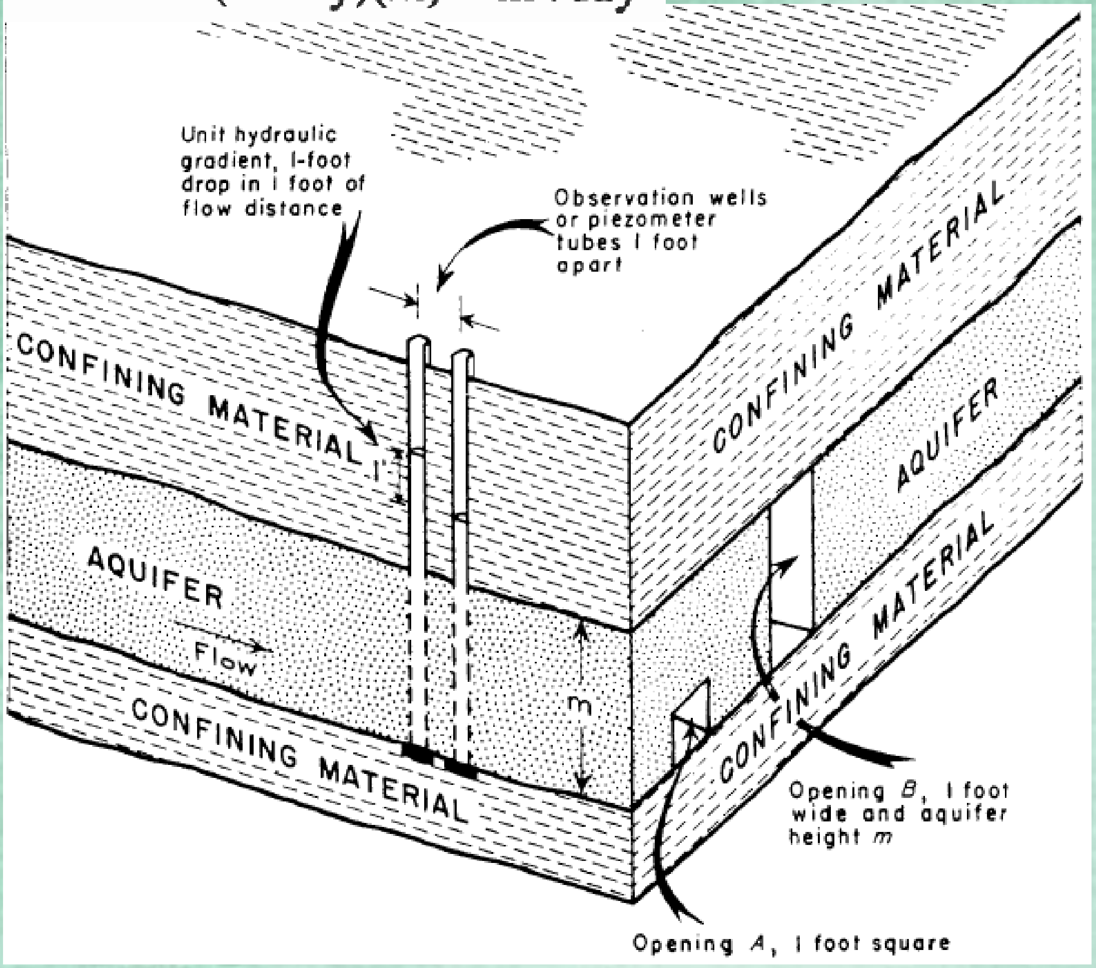




قابلیت انتقال

- به حاصلضرب هدایت هیدرولیکی در ضخامت آبخوان، گذردهی یا قابلیت انتقال می‌گویند. گاهی در معادلات جریان آب زیرزمینی از این پارامتر استفاده می شود.

$$T = Kb = (m/day)(m) = m^2/day$$



$$T_x = K_x .\ b$$

$$T_y = K_y .\ b$$

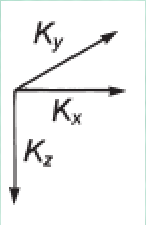
دیاگرام مفهومی هدایت هیدرولیکی و قابلیت انتقال

ناهمگنی و ناهمسانگردی در آبخوانها

- هدایت هیدرولیکی یک بردار است (در واقع یک تانسور است) بنابراین دارای مولفه هایی در جهات اصلی مختصات خواهد بود که با Kx, Ky, Kz نشان داده می شوند. بر این اساس معادله دارسی را می توان به شکل زیر تعمیم داد:

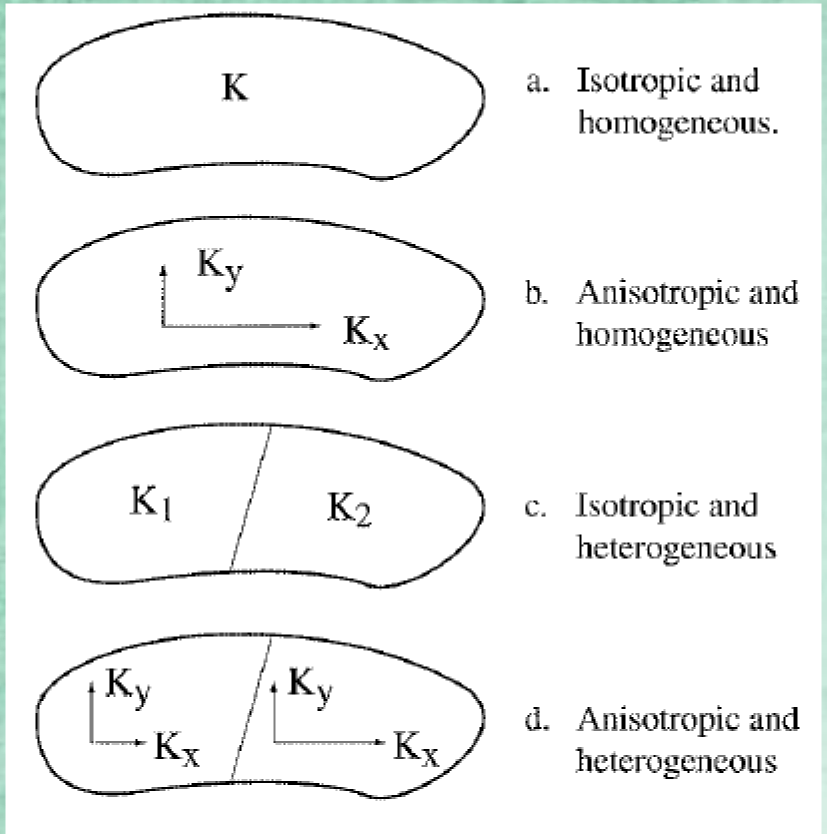
$$\vec{q} = q_x \vec{i} + q_y \vec{j} + q_z \vec{k}$$

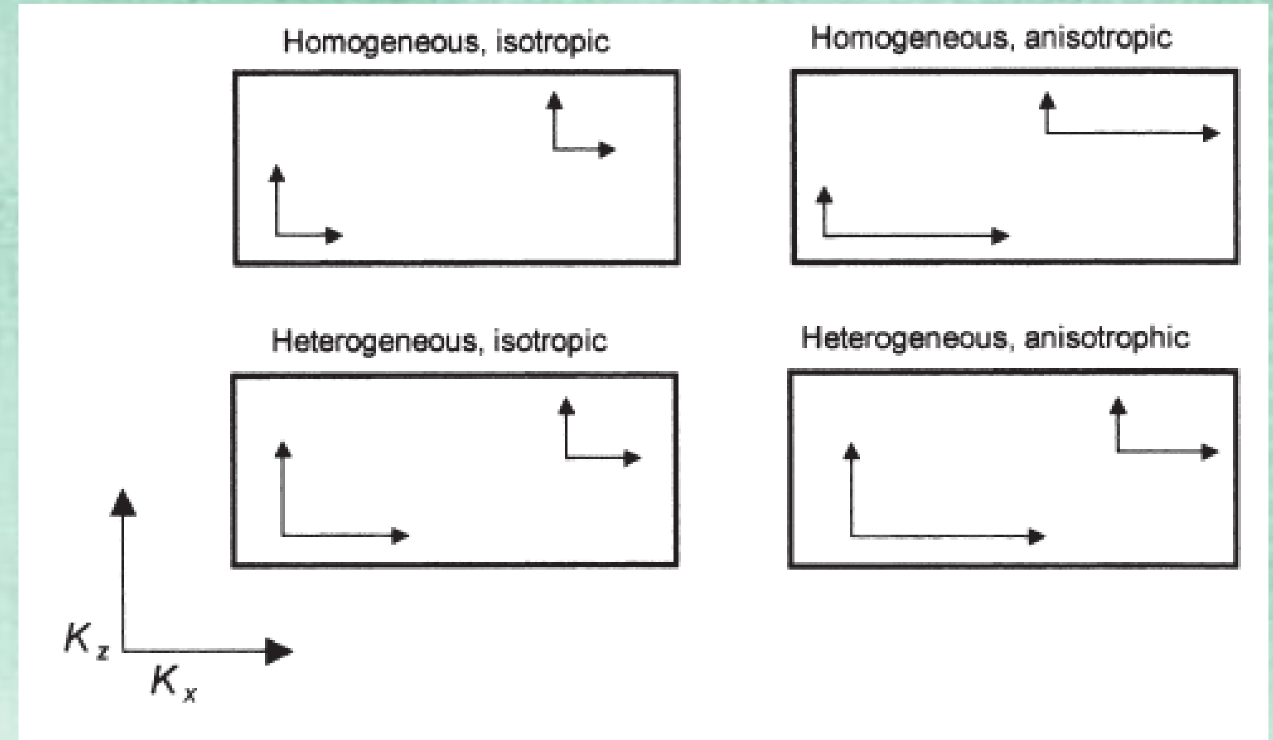
$$q_x = K_x \frac{\partial h}{\partial x}; \quad q_y = K_y \frac{\partial h}{\partial y}; \quad q_z = K_z \frac{\partial h}{\partial z}$$



- اگر هدایت هیدرولیکی در تمام نقاط آبخوان یکسان باشد، آبخوان همگن است. در غیر اینصورت ناهمگن است.
- اگر هدایت هیدرولیکی در تمام جهات درآبخوان یکسان باشد، آبخوان همسانگرد است. در غیر اینصورت ناهمسانگرد است.
- بنابراین در حالت کلی چهار نوع آبخوان از نظر همگنی و همسانگردی داریم (شکل ۳).
- نکته بسیار مهم که از این بحث نتیجه می شود این است که هدایت هیدرولیکی یک بردار (در اصل یک تانسور) است و مقدار آن در جهت های مختلف متفاوت است. در عمل معمولاً از همان جهت های محورهای مختصات دکارتی بدین منظور استفاده می شود و Kx, Ky, Kz خواهیم داشت.

ناهمگنی و ناهمسانگردی در آبخوانها

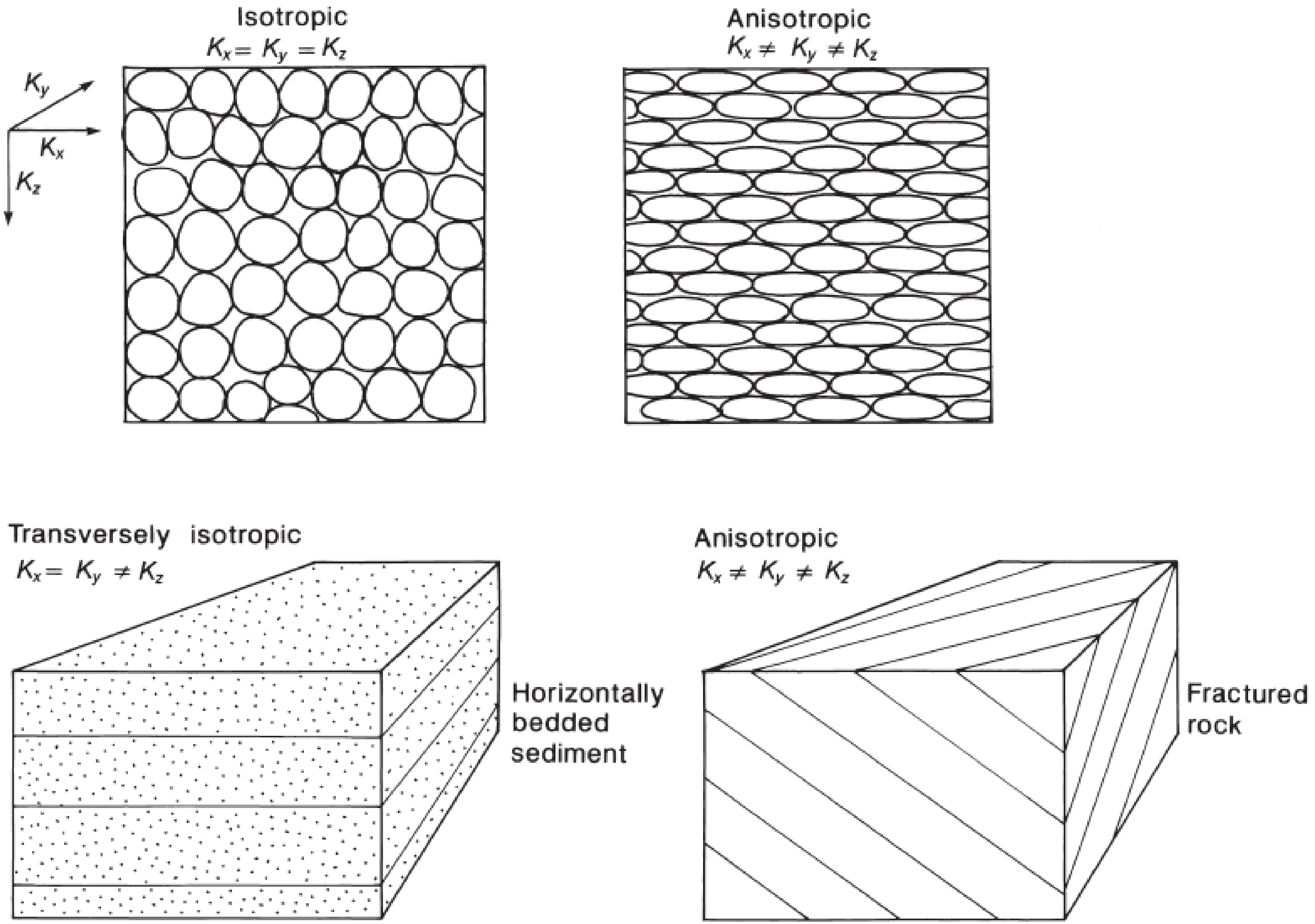




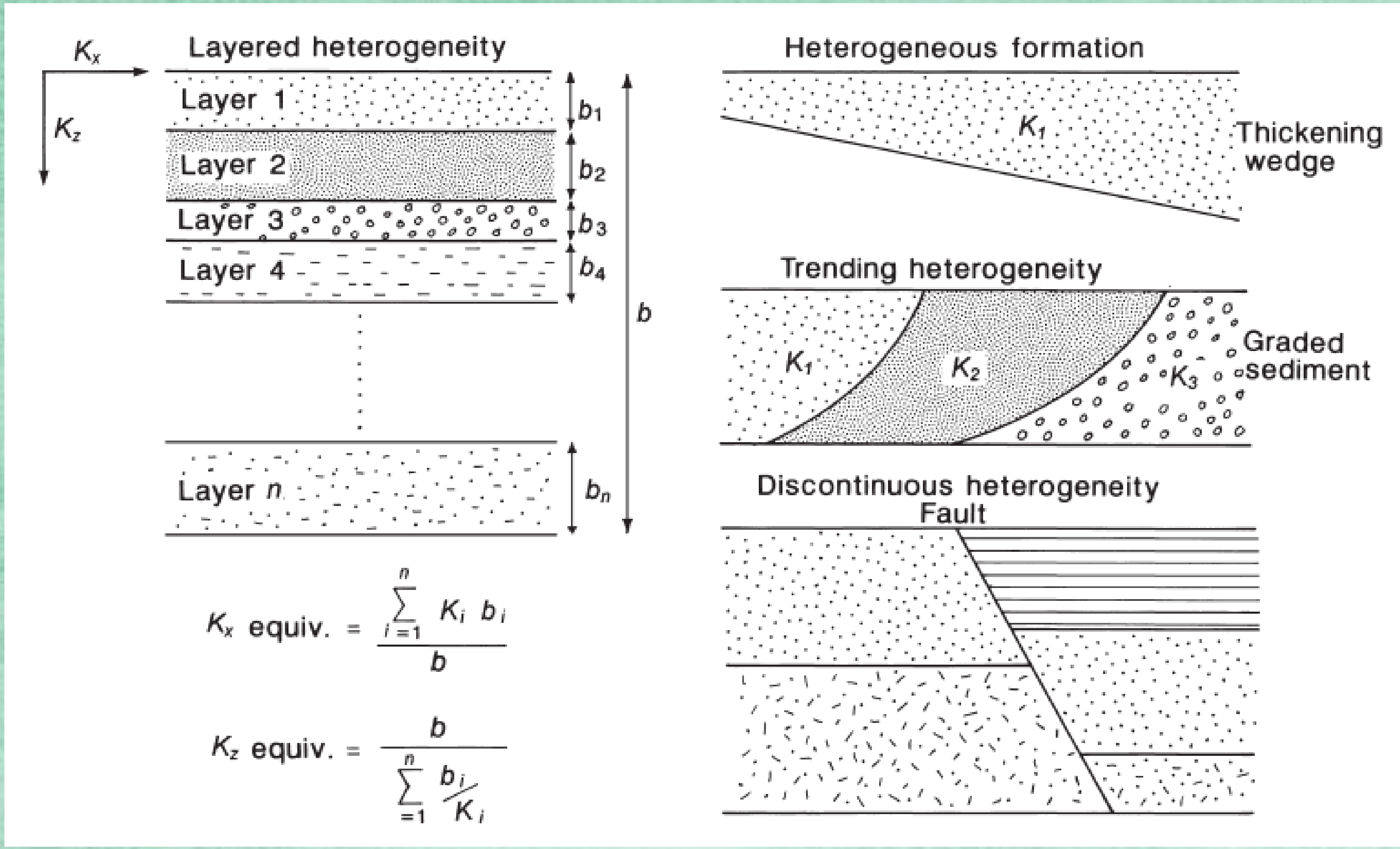
انواع آبخوان از نظر همگنی و همسانگردی

(Ref.: The Handbook of Groundwater Engineering, Delleur, 2007)

ناهمگنی و ناهمسانگردی در آبخوان‌ها



(Ref.: Hydrogeology Principles and Practice; Kevin M. Hiscock, 2005)



ناهمگنی در آبخوان ها

(Ref.: Hydrogeology Principles and Practice; Kevin M. Hiscock, 2005)

در یک لایه آبدار هدایت هیدرولیکی ۲۶متر بر روز، شیب هیدرولیکی ۰/۴، تخلخل ۰/۳۵و متوسط قطر ذرات خاک ۰/۰۴سانتی متر است. مطلوب است: الف) سرعت دارسی و سرعت واقعی ب) اگر لزجت دینامیک ۰/۰۰۱ نیوتن بر مترمربع بر ثانیه و چگالی آب ۱۰۰۰کیلوگرم بر مترمکعب باشد. آیا قانون دارسی در این لایه آبدار کاربرد دارد. ج) حداکثر سرعت جریان را در حالت آرام به دست آورید؟

$$V = Ki = ۲۶ \times ۰/۴ = ۱۰/۴ \, {}^{m}/_{day} = ۱/۲ \times ۱۰^{-۴} \, {}^{m}/_{sec}$$

$$V_s = \frac{V}{n} = \frac{۱/۲ \times ۱۰^{-۴}}{۰/۳۵} = ۲۹/۷۱ \, {}^{m}/_{day}$$

$$R_e = \frac{\rho V D}{\mu} = \frac{۱۰۰۰ \times ۱/۲ \times ۱۰^{-۴} \times ۰/۰۰۰۴}{۰/۰۰۱} = ۰/۰۴۸ < ۱$$

چون عدد رینولدز کوچکتر از یک است پس قانون دارسی معتبر است.
ج) حداکثر سرعت جریان در حالت آرام زمانی است که عدد رینولدز برابر با یک باشد

$$R_e = \frac{\rho V D}{\mu} \rightarrow ۱ = \frac{۱۰۰۰ \times V_{max} \times ۰/۰۰۰۴}{۰/۰۰۱} \rightarrow V_{max} = ۰/۰۰۲۵ \, {}^{m}/_{sec} = ۲۱۶ \, {}^{m}/_{day}$$

ناهمگنی و ناهمسانگردی در آبخوانها

- محاسبه K معادل در آبخوانهای ناهمگن لایه ای. جهت جریان موازی لایه ها.

$$Q=\sum_{i=1}^{3}Q_i;\quad \frac{\partial h}{\partial x}=Cte.$$

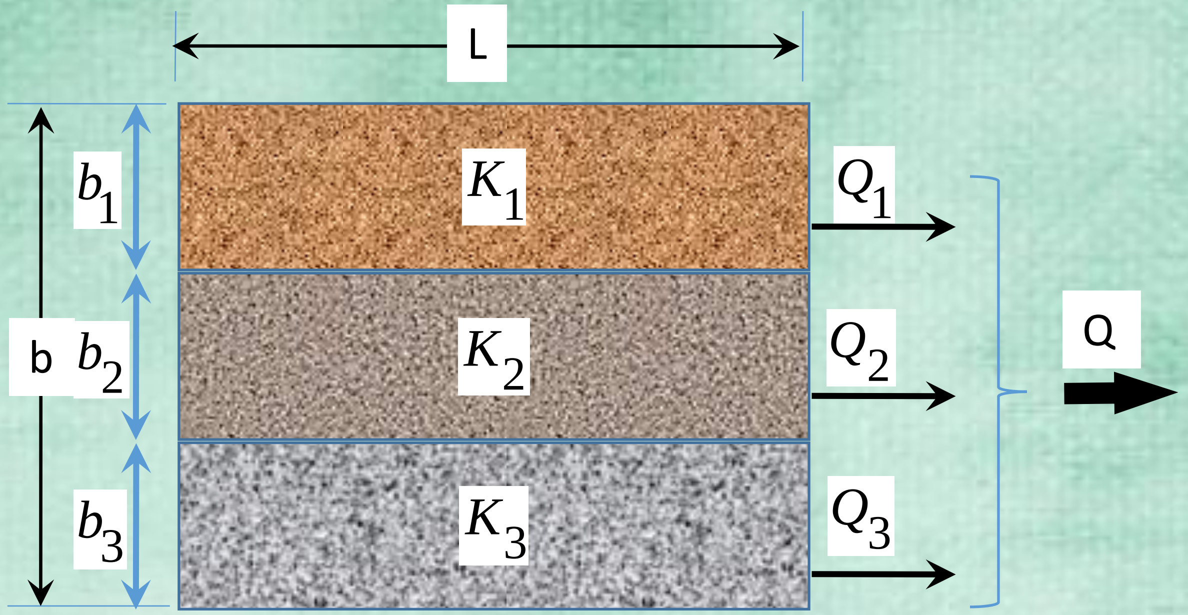
$$Q=\sum_{i=1}^{3}(-b_iK_i\frac{\partial h}{\partial x})$$

$$Q=-\frac{h_2-h_1}{L}\sum_{i=1}^{3}(b_iK_i)$$

$$Q=-\frac{h_2-h_1}{L}b\overline{K}$$

$$\sum_{i=1}^{3}b_i=b$$

$$\sum_{i=1}^{3}b_iK_i=b\overline{K}$$



$$\overline{K}^{Parallel}=\frac{1}{b}\sum_{i=1}^{3}(b_iK_i)$$

شکل ۳-۱۰-۱- K معادل در جریان به موازات لایه ها

ناهمگنی و ناهمسانگردی در آبخوانها

- محاسبه K معادل در آبخوانهای ناهمگن لایه ای. جهت جریان عمود بر لایه ها.

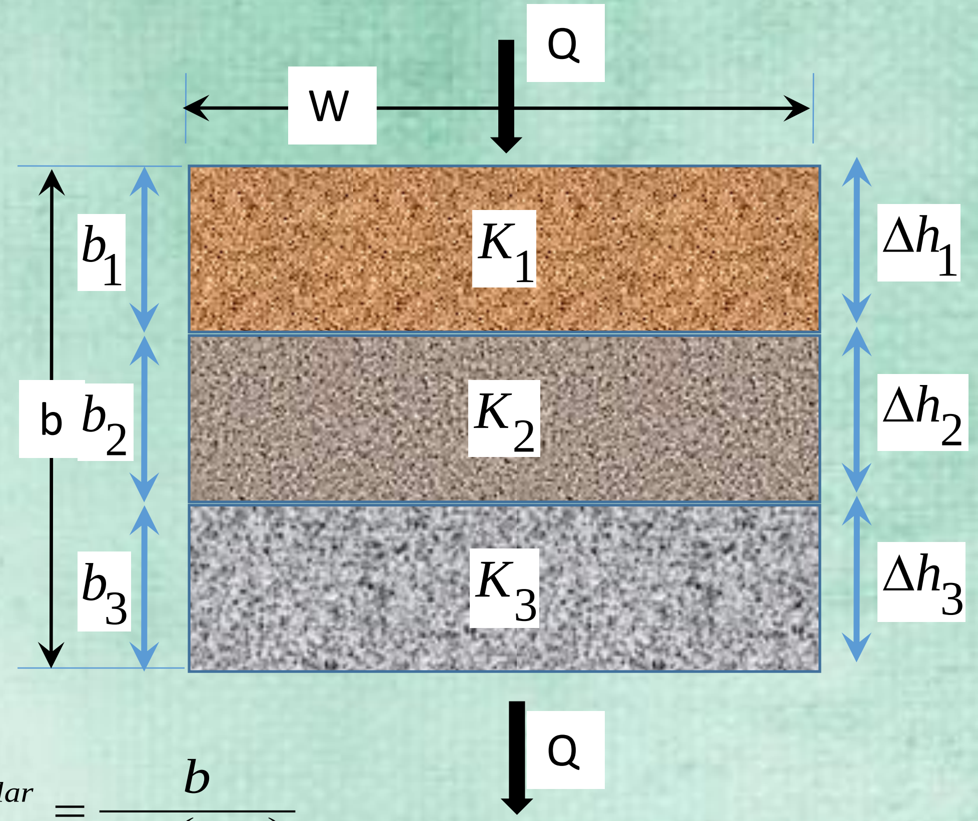
$$Q = -K_i W \frac{\Delta h_i}{b_i}$$

$$\Delta h_i = -\frac{b_i Q}{K_i W} = -\frac{Q}{W}\sum_{i=1}^{3}\left(\frac{b_i}{K_i}\right)$$

$$Q = -\overline{K} W \frac{\Delta h}{b}$$

$$\Delta h = -\frac{Q}{W}\frac{b}{\overline{K}}$$

$$\frac{b}{\overline{K}} = \sum_{i=1}^{3}\left(\frac{b_i}{K_i}\right)$$



$$\overline{K}^{Perpendicular} = \frac{b}{\sum_{i=1}^{3}\left(\frac{b_i}{K_i}\right)}$$

شکل ۳-۱۰-۲- K معادل در جریان عمود بر لایه ها

An unconfined aquifer consists of three horizontal layers, each individually isotropic. The top layer has a thickness of 10 m and a hydraulic conductivity of 11.6 m/day. The middle layer has a thickness of 4.4 m and a hydraulic conductivity of 4.5 m/day. The bottom layer has a thickness of 6.2 m and a hydraulic conductivity of 2.2 m/day. Compute the equivalent horizontal and vertical hydraulic conductivities.

Equation 3.4.5 is used to compute the equivalent horizontal hydraulic conductivity:

$$K_x = \frac{K_1 z_1 + K_2 z_2 + K_3 z_3}{z_1 + z_2 + z_3}$$

$$= \frac{(11.6 \text{ m/day})(10 \text{ m}) + (4.5 \text{ m/day})(4.4 \text{ m}) + (2.2 \text{ m/day})(6.2 \text{ m})}{(10 \text{ m} + 4.4 \text{ m} + 6.2 \text{ m})} = 7.25 \text{ m/day}$$

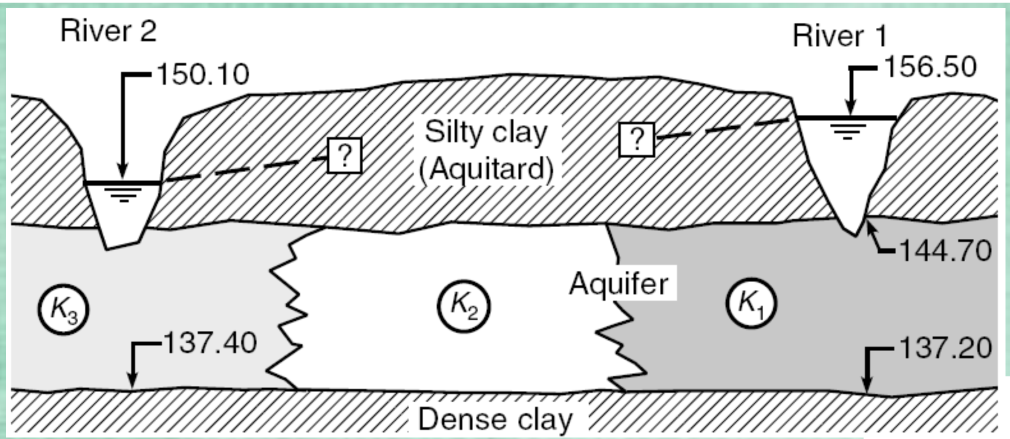
The equivalent vertical hydraulic conductivity is computed using Equation 3.4.12:

$$K_z = \frac{z_1 + z_2 + z_3}{\frac{z_1}{K_1} + \frac{z_2}{K_2} + \frac{z_3}{K_3}}$$

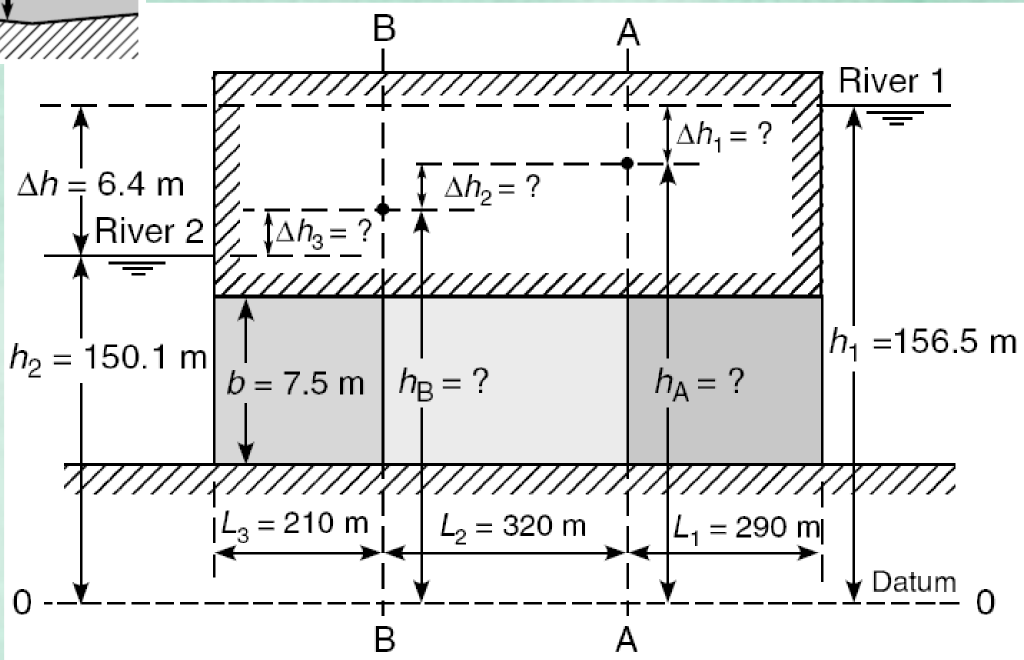
$$= \frac{10 \text{ m} + 4.4 \text{ m} + 6.2 \text{ m}}{\frac{10 \text{ m}}{11.6 \text{ m/day}} + \frac{4.4 \text{ m}}{4.5 \text{ m/day}} + \frac{6.2 \text{ m}}{2.2 \text{ m/day}}} = 4.42 \text{ m/day}$$

Note that the equivalent hydraulic conductivities above are computed based on the assumption that each layer is individually isotropic, that is, $K_x = K_z$ in each layer. ■

Calculate the groundwater flow per unit width of a confined aquifer in the section between two parallel rivers shown in Figure 10.3. The aquifer can be divided into three homogeneous areas with different hydraulic conductivities: $K_1 = 6.2 \times 10^{-5}$ m/s, $K_2 = 7.9 \times 10^{-4}$ m/s, $K_3 = 4.1 \times 10^{-4}$ m/s. Draw a calculation scheme showing all parameters necessary for the solution. Determine the position of the hydraulic head (potentiometric) surface on the cross section. Determine the "equivalent hydraulic conductivity" and compare it with the average hydraulic conductivity. Also compare the results of groundwater flow calculation using these two hydraulic conductivities with the result obtained using the original calculation scheme.



$K_1 = 6.2 \times 10^{-5}$ m/s, $K_2 = 7.9 \times 10^{-4}$ m/s, $K_3 = 4.1 \times 10^{-4}$ m/s



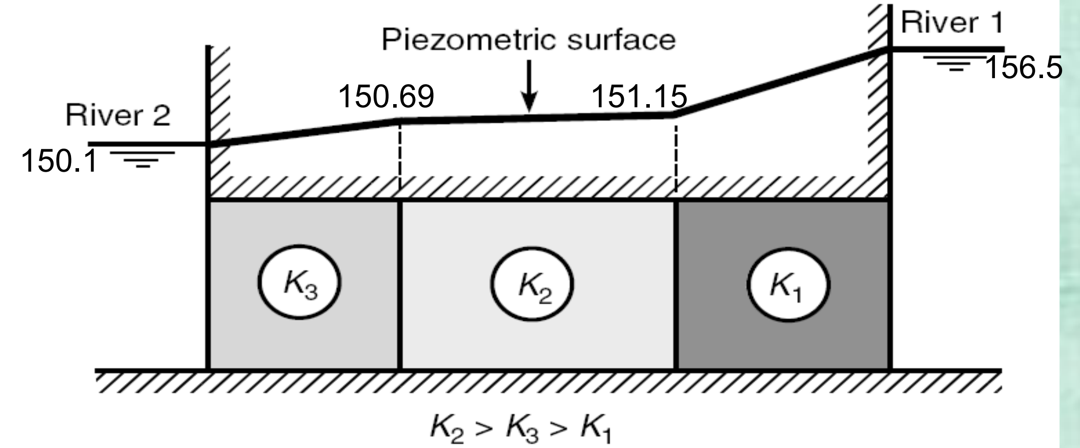
$$K_{eqv} = \frac{L}{\sum_{i=1}^{i=3} \frac{L_i}{K_i}}$$

$$K_{eqv} = \frac{290\ \text{m} + 320\ \text{m} + 210\ \text{m}}{\frac{290\ \text{m}}{6.2\times10^{-5}\ \text{m/s}} + \frac{320\ \text{m}}{7.9\times10^{-4}\ \text{m/s}} + \frac{210\ \text{m}}{4.1\times10^{-4}\ \text{m/s}}}$$

$$K_{eqv} = 1.47 \times 10^{-4}\ \text{m/s}$$

Δh=156.5-150.1=6.4 m;
L=L1+L2+L3=290+320+210=820 m
i=Δh/L=6.4/820=0.007805
q=Keq.b.i=1.47×7.5×1.47×10^-4

$$q = 8.58 \times 10^{-6}\ \text{m}^2/\text{s}$$



$$q = bK\frac{h_1 - h_2}{L}$$

$$q = T\frac{h_1 - h_2}{L}$$

$$h_1 - h_2 = \Delta h = \frac{qL}{T}$$

$$\Delta h_1 = \frac{q_1 L_1}{T_1}$$

$$\Delta h_2 = \frac{q_2 L_2}{T_2}$$

$$\Delta h_3 = \frac{q_3 L_3}{T_3}$$

$$\Delta h = \Delta h_1 + \Delta h_2 + \Delta h_3$$

$$\Delta h = \sum_{i=1}^{i=3} \Delta h_i$$

$$q_1 = q_2 = q_3$$

$$\Delta h_1 = \frac{q_1 L_1}{T_1} = 5.35\ \text{m}$$

$$\Delta h_2 = \frac{q_2 L_2}{T_2} = 0.46\ \text{m}$$

$$\Delta h_3 = \frac{q_3 L_3}{T_3} = 0.59\ \text{m}$$

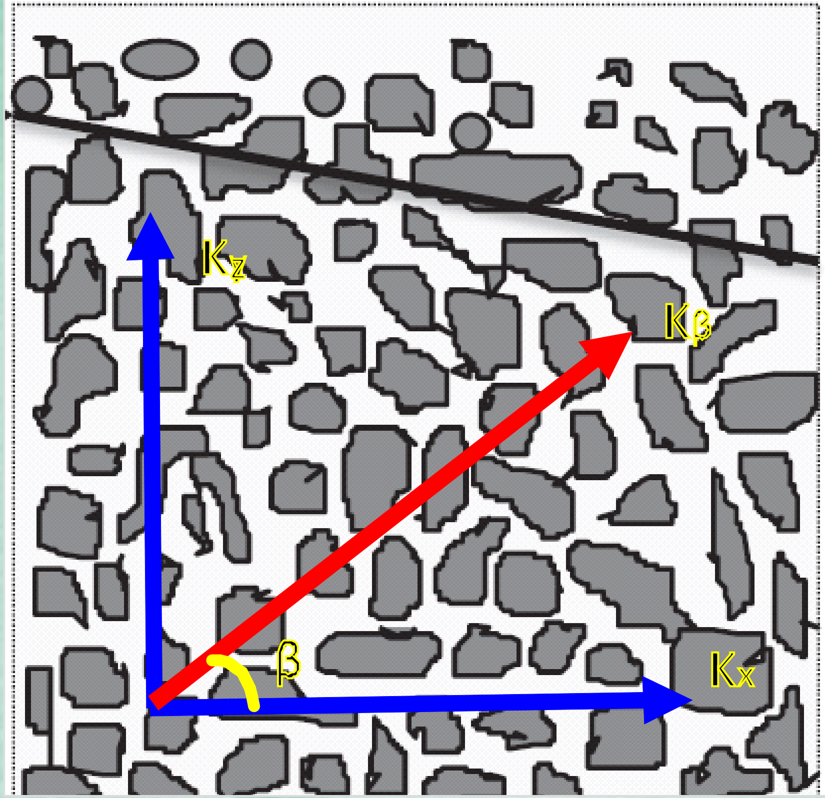
$$h_A = h_1 - \Delta h_1 = 151.15\ \text{m}$$

$$h_B = h_A - \Delta h_2 = 150.69\ \text{m}$$

- اگر هدایت هیدرولیکی در جهت افقی Kx و در جهت عمودی Ky باشد، در جهت دلخواه β برابر است با:

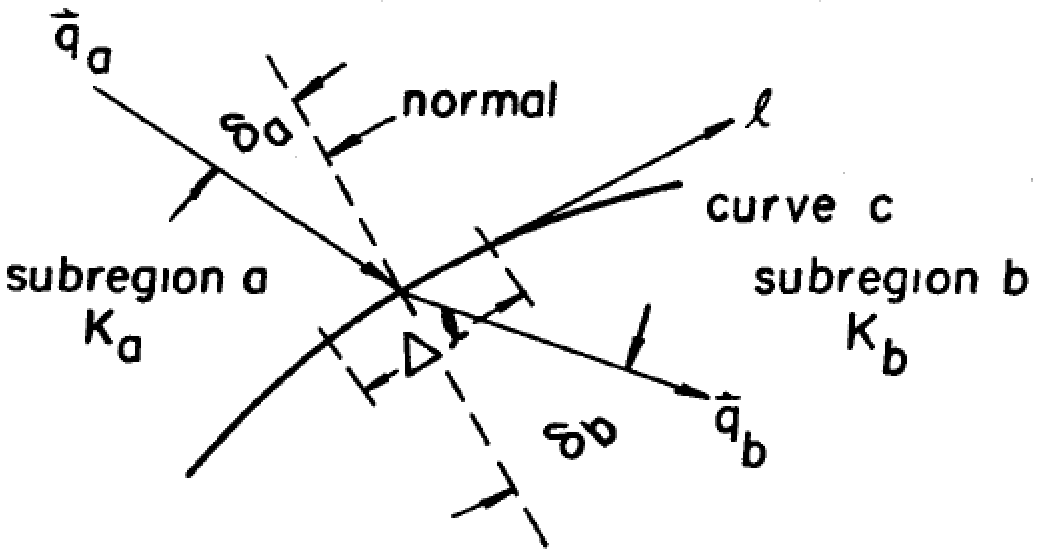
$$K_{\beta} = \frac{K_x . K_z}{K_x . \sin^2 \beta + K_z . \cos^2 \beta}$$

$$\frac{1}{K_{\beta}} = \frac{\cos^2 \beta}{K_x} + \frac{\sin^2 \beta}{K_z}$$



شکل ۳-۱۱- K در جهت دلخواه

مسیر جریان در محل تقاطع دو لایه با هدایت هیدرویکی متفاوت



شکل ۳-۱۲

Figure 3-8. Refraction of the velocity vector at a boundary between different homogeneous media.

$$\frac{K_a}{K_b} = \frac{\tan \delta_a}{\tan \delta_b} \quad (3\text{-}30)$$

EXAMPLE 3-8

The Darcy velocity is incident on the interface between coarse and fine textured materials with hydraulic conductivities $K_a = 1.6\times10^{-3}$ cm/s and $K_b = 1.2\times10^{-4}$ cm/s. Flow occurs from the coarse material into the fine and makes an angle $\delta = 30°$ with the normal to the interface. Calculate the angle δ_b in the fine textured material.

Solution:

From Eq. 3-30

$$\tan \delta_b = \frac{K_b}{K_a} \tan \delta_a = \frac{1.2\times10^{-4}}{1.6\times10^{-3}} \tan 30° = 0.0433 \quad .$$

Hence

$$\delta_b = 2.5° \quad .$$

It is observed that the flow in the fine layer is nearly normal to the interface.

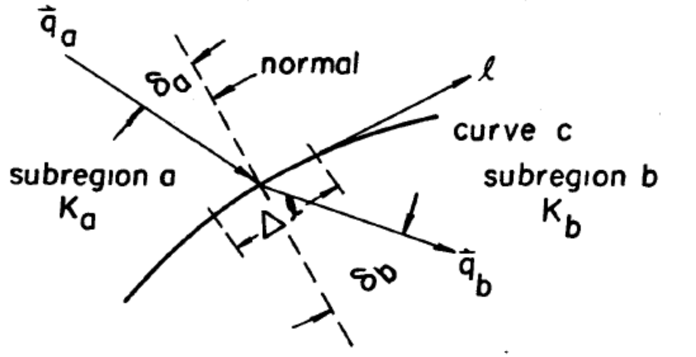


Figure 3-8. Refraction of the velocity vector at a boundary between different homogeneous media.

EQUATIONS OF MOTION IN AQUIFERS: GENERALIZATION OF DARCY'S LAW

Darcy Velocities in Two- and Three-Dimensional Cartesian Coordinates

Equations of motion is given in the Cartesian coordinates for the special case where the principal directions of anisotropy coincide with *x*-, *y*-, and *z*-coordinate axes. In other words, the Darcy velocity vector has three components that are aligned with the *x*-, *y*-, and *z*-coordinate axis. These components are shown on an infinitesimal parallelepiped volume that represents a point in the flow field (Figure 4-13).

Equation (4-19) is the one-dimensional differential form of Darcy's law and it states that the flow rate through porous materials in any direction is proportional to the negative rate of change in the hydraulic head in that direction. The negative sign in the equation implies that the fluid move in the direction of decreasing hydraulic head.

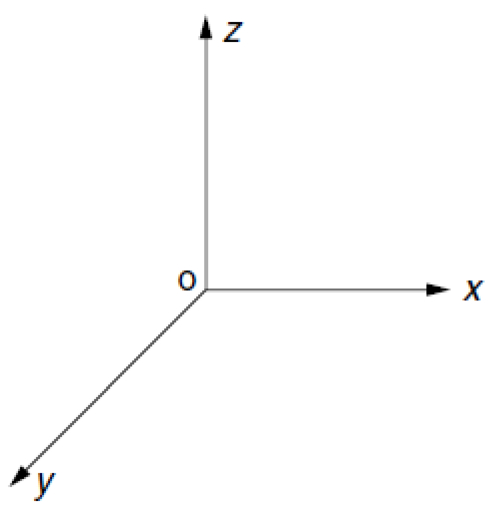
Darcy's law was extended to three-dimensional cases (e.g., Polubarinova-Kochina, 1962; Hantush, 1964; Bear, 1972, 1979; Freeze and Cherry, 1979). The general form of Darcy's law in a nonhomogeneous and anisotropic porous medium, with the principal axes of the hydraulic conductivity tensor parallel to the Cartesian coordinate axes in vector form is

$$\mathbf{q} = q_x\mathbf{i} + q_y\mathbf{j} + q_z\mathbf{k} \tag{4-42}$$

where $\mathbf{q}$ is the Darcy velocity vector, and q_x, q_y, and q_z and **i**, **j**, and **k** are the Darcy velocity components and the unit vectors in the *x*, *y*, and *z* directions, respectively. The general, spatially

(Ref.: Applied Flow and Solute Transport Modeling in Aquifers, Batu, 2006)

شکل ۳-۱۳



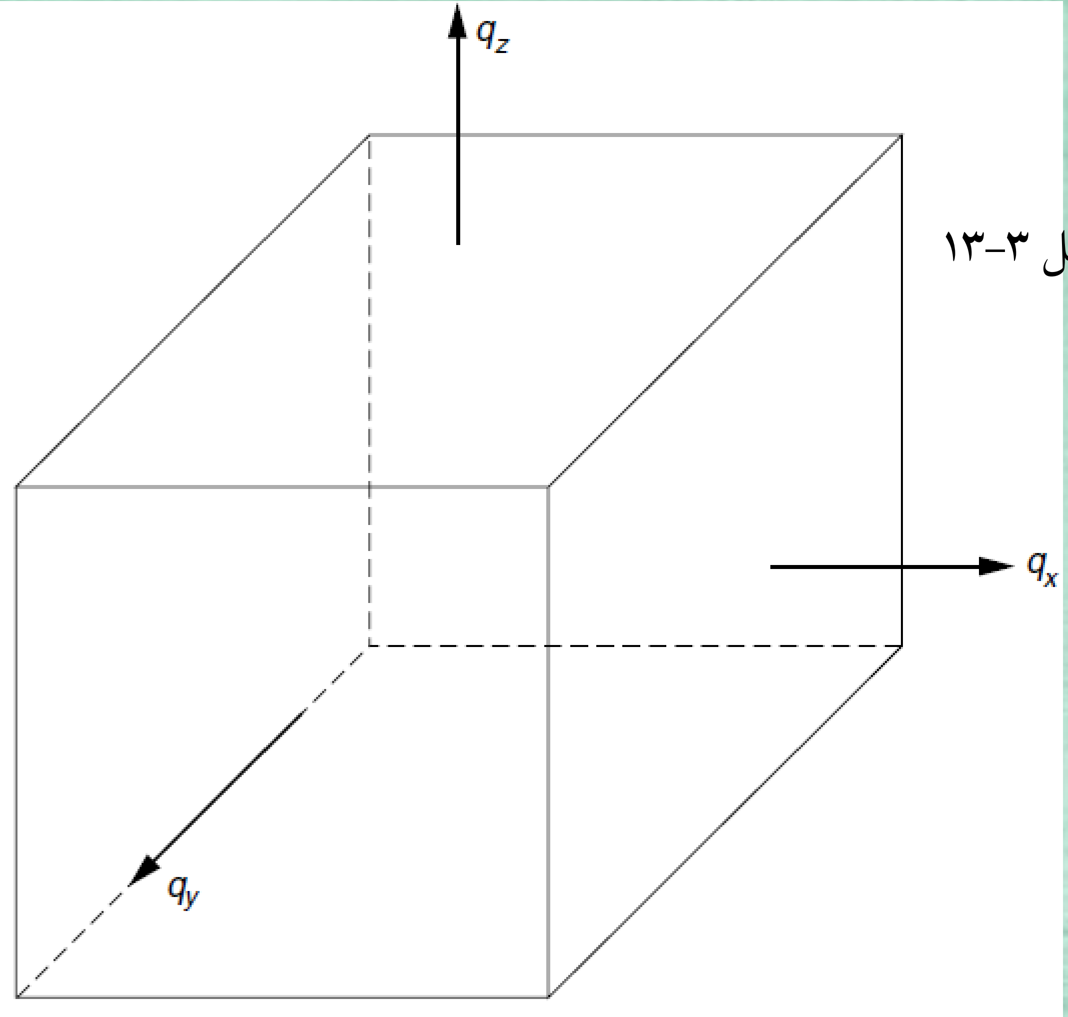


FIGURE 4-13 Darcy velocity components under the condition that the principal directions of anisotropy concide with *x*, *y*, and *z* directions of the coordinate axes.

(Ref.: Applied Flow and Solute Transport Modeling in Aquifers, Batu, 2006)

خطوط هم پتانسیل و خطوط جریان

- **تابع پتانسیل سرعت**

در آب زیرزمینی به حاصل‌ضرب $\Phi=Kh$ تابع پتانسیل سرعت (یا پتانسیل سرعت) گویند.

معادله جریان دوبعدی (در پلان) در یک آبخوان تحت فشار همگن همسانگرد در رژیم دایم به صورت

$$\nabla^2 h = \frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial y^2} = 0$$

به دست آمد که همان معادله لاپلاس است. با توجه به تعریف بالا همچنین داریم:

$$\nabla^2 \Phi = \frac{\partial^2 \Phi}{\partial x^2} + \frac{\partial^2 \Phi}{\partial y^2} = 0$$

توابع $\Phi(x,y)$ که در رابطه بالا صدق کنند را توابع هارمونیک می‌نامند. منحنی‌هایی در صفحه x-y با $\Phi(x,y)=Cte$ را کانتورها یا خطوط هم‌پتانسیل گویند. (در صفحات قائم x-z و y-z هم به طور مشابه تعریف می‌شود.) یک کانتور هم‌پتانسیل مکان هندسی نقاطی است که در سرتاسر آن هد پیزومتریک مقداری ثابت دارد.

خطوط هم پتانسیل و خطوط جریان

- **تابع پتانسیل سرعت و تابع جریان**

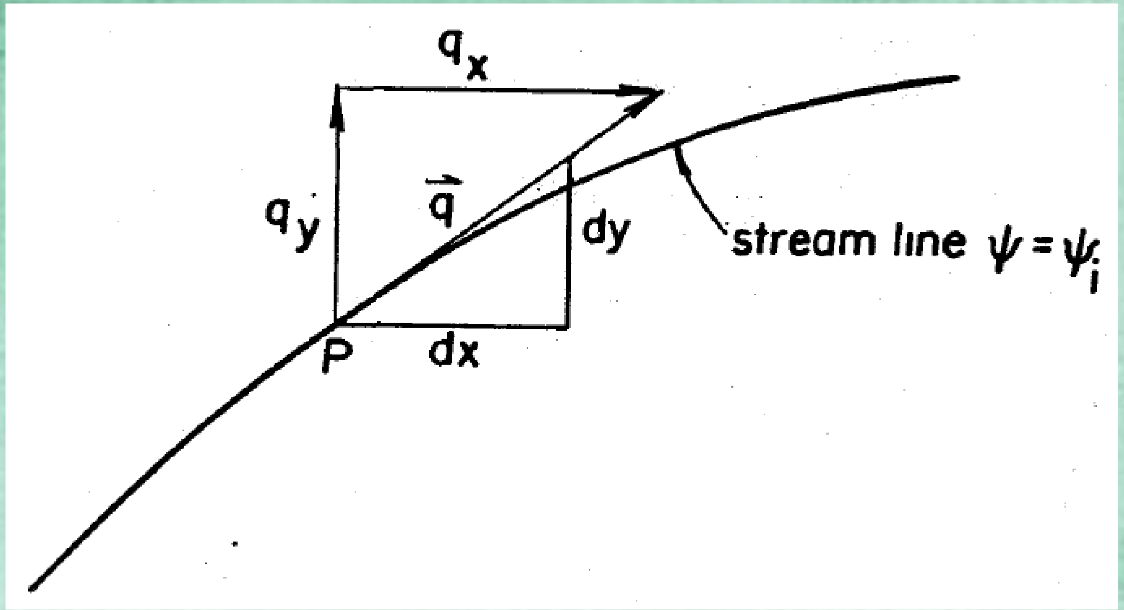
در یک آبخوان همگن همسانگرد مشتق Φ در هر جهت، مؤلفه سرعت دارسی در آن جهت است:

$$q_l = V_l = -K\frac{\partial h}{\partial l} = -\frac{\partial (Kh)}{\partial l} = -\frac{\partial \Phi}{\partial l}$$

اگر جهت l را هم‌جهت با یک خط هم‌پتانسیل (که دارای مقداری ثابت است) در یک نقطه دلخواه P در نظر بگیریم، مشتق آن برابر صفر بوده و بنابراین نتیجه می‌شود که سرعت دارسی $\vec{q}$ در نقطه P فاقد مولفه‌ای مماس بر خط هم‌پتانسیل است.

تابع جریان $\psi(x,y)$ بر اساس مفهوم بالا اینگونه تعریف می‌شوند که عبارتند از خانواده‌ای از منحنی‌ها که همه‌جا مماس بر بردار سرعت دارسی $\vec{q}$ هستند. مکان هندسی نقاطی با $\psi(x,y)=Cte$ را خطوط جریان گویند. رابطه بین تابع جریان و مؤلفه‌های بردار سرعت را می‌توان با استفاده از خط جریان شکل ۴-۱ نشان داد. از شکل ۴-۱ شیب خط جریان در نقطه P برابر $(dy/dx)_\psi$ و برابر نسبت مولفه بردار سرعت در جهات y و x یا (q_y/q_x) است:

خطوط هم پتانسیل و خطوط جریان



شکل ۴-۱- رابطه بین مولفه‌های بردار سرعت و خط جریان

$$\frac{q_y}{q_x} = \left(\frac{dy}{dx} \right)_{\psi} \quad ,$$

or $\qquad\qquad\qquad\qquad\qquad\qquad$ (4-3)

$$q_y dx - q_x dy = 0 \quad .$$

خطوط هم پتانسیل و خطوط جریان

For $\psi(x,y) = \psi_1$, where ψ_1 is the constant representing the particular streamline through P, we have

$$d\psi_1 = \frac{\partial \psi}{\partial x} dx + \frac{\partial \psi}{\partial y} dy = 0 \quad . \qquad (4\text{-}4)$$

The coefficients of dx and dy in Eqs. 4-3 and 4-4 must be equal since dx and dy are the same in both equations. Therefore,

$$q_x = -\frac{\partial \psi}{\partial y} \quad , \qquad (4\text{-}5)$$

and

$$q_y = \frac{\partial \psi}{\partial x} \quad . \qquad (4\text{-}6)$$

It can now be easily proven that streamlines and equipotential lines intersect at right angles. At point P on an equipotential line $\Phi(x,y) = \Phi_1$,

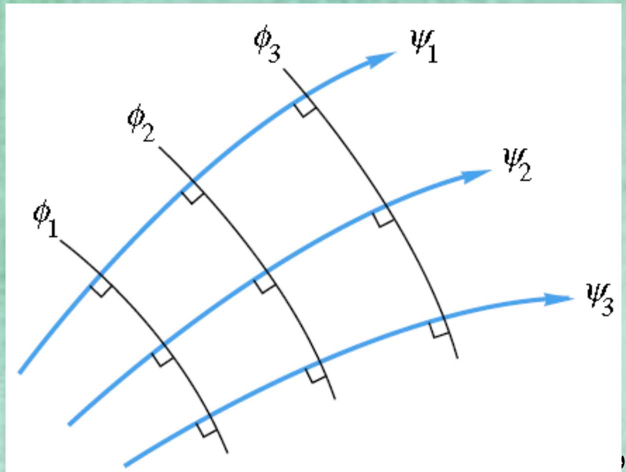
$$d\Phi_1 = \frac{\partial \Phi}{\partial x} dx + \frac{\partial \Phi}{\partial y} dy = 0 \quad , \qquad (4\text{-}7)$$

so that

$$\left(\frac{dy}{dx} \right)_\Phi = -\frac{\partial \Phi / \partial x}{\partial \Phi / \partial y} = -\frac{q_x}{q_y} \qquad (4\text{-}8)$$

where $(dy/dx)_\Phi$ is the slope of the equipotential line.

خطوط هم پتانسیل و خطوط جریان



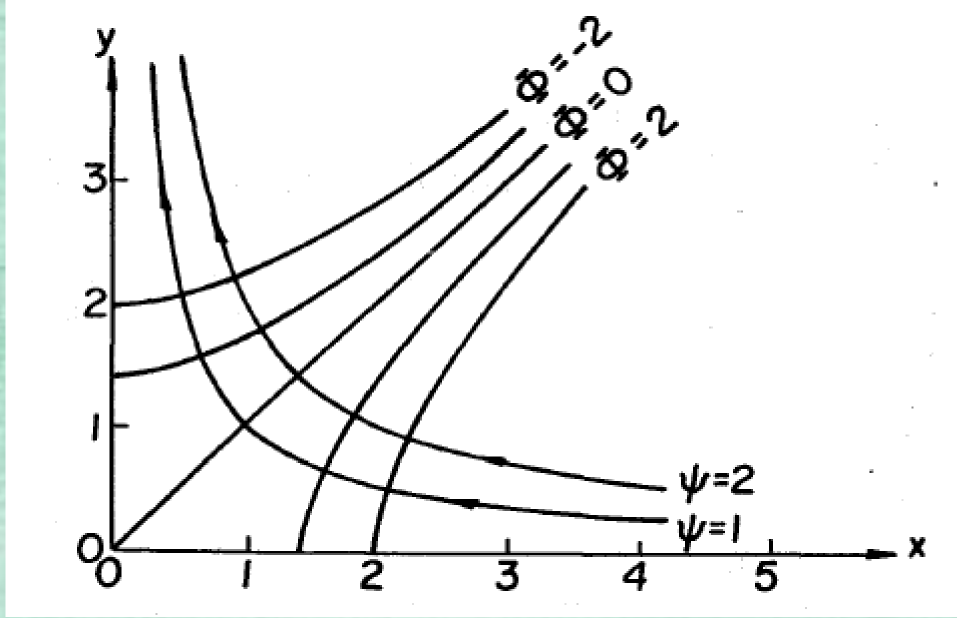
از معادلات قبل می‌توان نتیجه گرفت که:

$$\frac{\partial \Phi}{\partial x} = \frac{\partial \psi}{\partial y}$$

$$\frac{\partial \psi}{\partial x} = -\frac{\partial \Phi}{\partial y}$$

معادلات بالا را معادلات کوشی-ریمان گویند و توابع پتانسیل و جریان را ب...

مثال: چندین خط جریان و خط هم‌پتانسیل را با تابع جریان $\psi = xy$ در ربع اول مختصات رسم کنید.



$$q_x = -\frac{\partial \Phi}{\partial x} \quad , \quad q_y = -\frac{\partial \Phi}{\partial y}$$

$$\frac{\partial \Phi}{\partial x} = x \quad , \quad \frac{\partial \Phi}{\partial y} = -y$$

$$\Phi = \frac{x^2}{2} + f(y) \quad , \quad \Phi = \frac{-y^2}{2} + g(x)$$

$$\Phi = \frac{1}{2}(x^2 - y^2)$$

and temporally variable forms of Darcy velocity components in the x, y, and z, directions, respectively, are

$$q_x = q_x(x, y, z, t) = -K_x(x, y, z)\frac{\partial h(x, y, z, t)}{\partial x} \tag{4-43a}$$

$$q_y = q_y(x, y, z, t) = -K_y(x, y, z)\frac{\partial h(x, y, z, t)}{\partial y} \tag{4-43b}$$

$$q_z = q_z(x, y, z, t) = -K_z(x, y, z)\frac{\partial h(x, y, z, t)}{\partial z} \tag{4-43c}$$

In the two-dimensional case, Eqs. (4-43a), (4-43b), and (4-43c), respectively, take the forms

$$q_x = q_x(x, y, t) = -K_x(x, y)\frac{\partial h(x, y, t)}{\partial x} \tag{4-44a}$$

$$q_y = q_y(x, y, t) = -K_x(x, y)\frac{\partial h(x, y, t)}{\partial x} \tag{4-44b}$$

$$q_z = 0 \tag{4-44c}$$

(Ref.: Applied Flow and Solute Transport Modeling in Aquifers, Batu, 2006)

With the assumption of constant principal hydraulic conductivities, in aquifer hydraulics, it is further assumed that the principal hydraulic conductivities in the horizontal plane are the same, i.e., $K_x(x, y, z) = K_y(x, y, z) = K_h$ and $K_z(x, y, z) = K_z$, and Eqs. (4-43a), (4-43b), and (4-43c), respectively, take the following forms:

$$q_x = q_x(x, y, z, t) = -K_h \frac{\partial h(x, y, z, t)}{\partial x} \tag{4-45a}$$

$$q_y = q_y(x, y, z, t) = -K_h \frac{\partial h(x, y, z, t)}{\partial y} \tag{4-45b}$$

$$q_z = q_z(x, y, z, t) = -K_z \frac{\partial h(x, y, z, t)}{\partial z} \tag{4-45c}$$

In Eq. (4-45c), K_v is often used instead of K_z.

(Ref.: Applied Flow and Solute Transport Modeling in Aquifers, Batu, 2006)

Darcy Velocities in Two- and Three-Dimensional Cylindrical Coordinates

Cylindrical coordinate system has wide applications in deriving analytical solutions for circular, vertical well hydraulics problems owing to the geometric shape of wells. A circular well generally has a finite radius and it is assumed that the central axis of the well coincides with the z-coordinate axis. The radial distance in a horizontal plane is measured from the vertical axis. Both the Cartesian and cylindrical coordinates are shown in Figure 4-14. Under extraction or injection condition from or to a circular well, a radial flow to or from the well occurs. Under this condition, the hydraulic conductivity K_r is the only horizontal hydraulic conductivity instead of K_x and K_y in the x and y directions, respectively. The vertical hydraulic conductivity K_z remains the same. Therefore, the equivalent forms of Eqs. (4-45) take the following forms:

$$q_r = q_r(r, z, t) = -K_r \frac{\partial h(r, z, t)}{\partial r} \tag{4-46a}$$

$$q_z = q_z(r, z, t) = -K_z \frac{\partial h(r, z, t)}{\partial z} \tag{4-46b}$$

It is apparent that the equivalent forms of Eq. (4-46a) in the Cartesian coordinates are Eqs. (4-45a) and (4-45b). Equation (4-46a) is based on the assumption that the principal direction of the horizontal hydraulic conductivity is in the radial direction K_r and its value is the same in all directions around the well. The direction of the vertical hydraulic conductivity is in the direction of the z coordinate.

(Ref.: Applied Flow and Solute Transport Modeling in Aquifers, Batu, 2006)

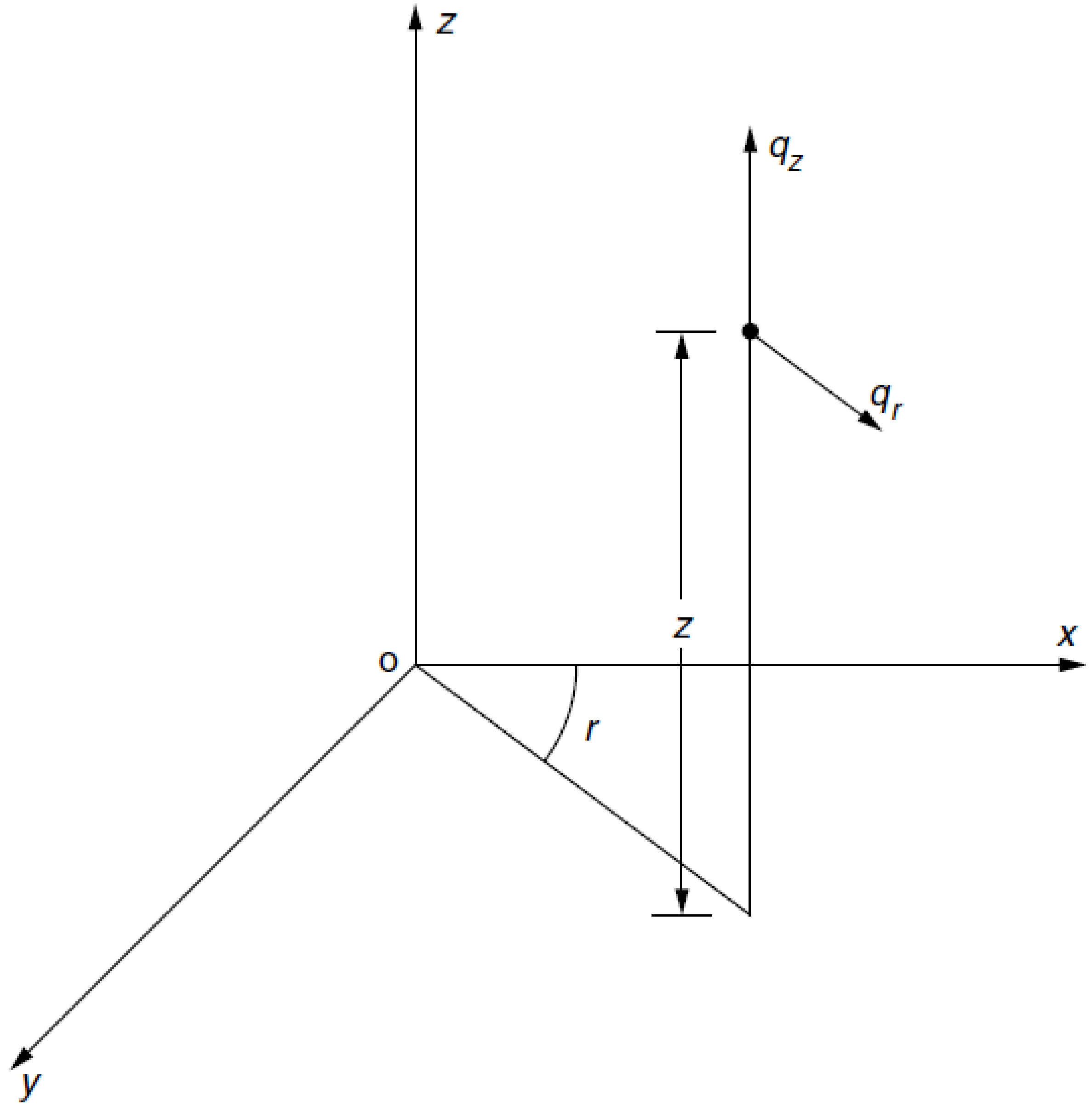


FIGURE 4-14 Darcy velocity components for the cylindrical coordinates system.

(Ref.: Applied Flow and Solute Transport Modeling in Aquifers, Batu, 2006)

4.3.2.2 Equations of Motion: Principal Directions of Anisotropy Do Not Coincide with the Directions of Coordinate Axes

Equations of motion will be written for the general case where the principal directions of anisotropy do not coincide with x-, y-, and z-coordinate axes. These components are shown on an infinitesimal parallelepiped volume, which represents a point in the flow field shown in Figure 4-15. The Darcy velocity components of the vector **q** given by Eq. (4-42) are (see, e.g., Bear, 1972; Domenico and Schwartz, 1990)

$$q_{xx} = -K_{xx}\frac{\partial h}{\partial x} - K_{xy}\frac{\partial h}{\partial y} - K_{xz}\frac{\partial h}{\partial z} \tag{4-47a}$$

$$q_{yy} = -K_{yx}\frac{\partial h}{\partial x} - K_{yy}\frac{\partial h}{\partial y} - K_{yz}\frac{\partial h}{\partial z} \tag{4-47b}$$

$$q_{zz} = -K_{zx}\frac{\partial h}{\partial x} - K_{zy}\frac{\partial h}{\partial y} - K_{zz}\frac{\partial h}{\partial z} \tag{4-47c}$$

where x, y, and z are the Cartesian coordinates and K_{xx}, K_{xy}, ..., K_{zz} are the nine constant components of the hydraulic conductivity tensor in the most general case. The nine components in the matrix form display a second-rank symmetric tensor known as the hydraulic conductivity tensor (Bear, 1972). In Eqs. (4-47), the first subscript indicates the direction perpendicular to the plane upon which the Darcy velocity vector acts and the second subscript indicates the direction of the Darcy velocity vector in that plane. If the principal directions of anisotropy coincide with x, y, and z directions of the coordinate axes, the six components K_{xy}, K_{xz}, K_{yx}, K_{yz}, K_{zx}, and K_{zy} become equal to zero, and Eqs. (4-47) reduces to Eqs. (4-43) for $K_x(x, y, z) = K_x$, $K_y\,(x, y, z) = K_y$, and $K_z\,(x, y, z) = K_z$.

(Ref.: Applied Flow and Solute Transport Modeling in Aquifers, Batu, 2006)

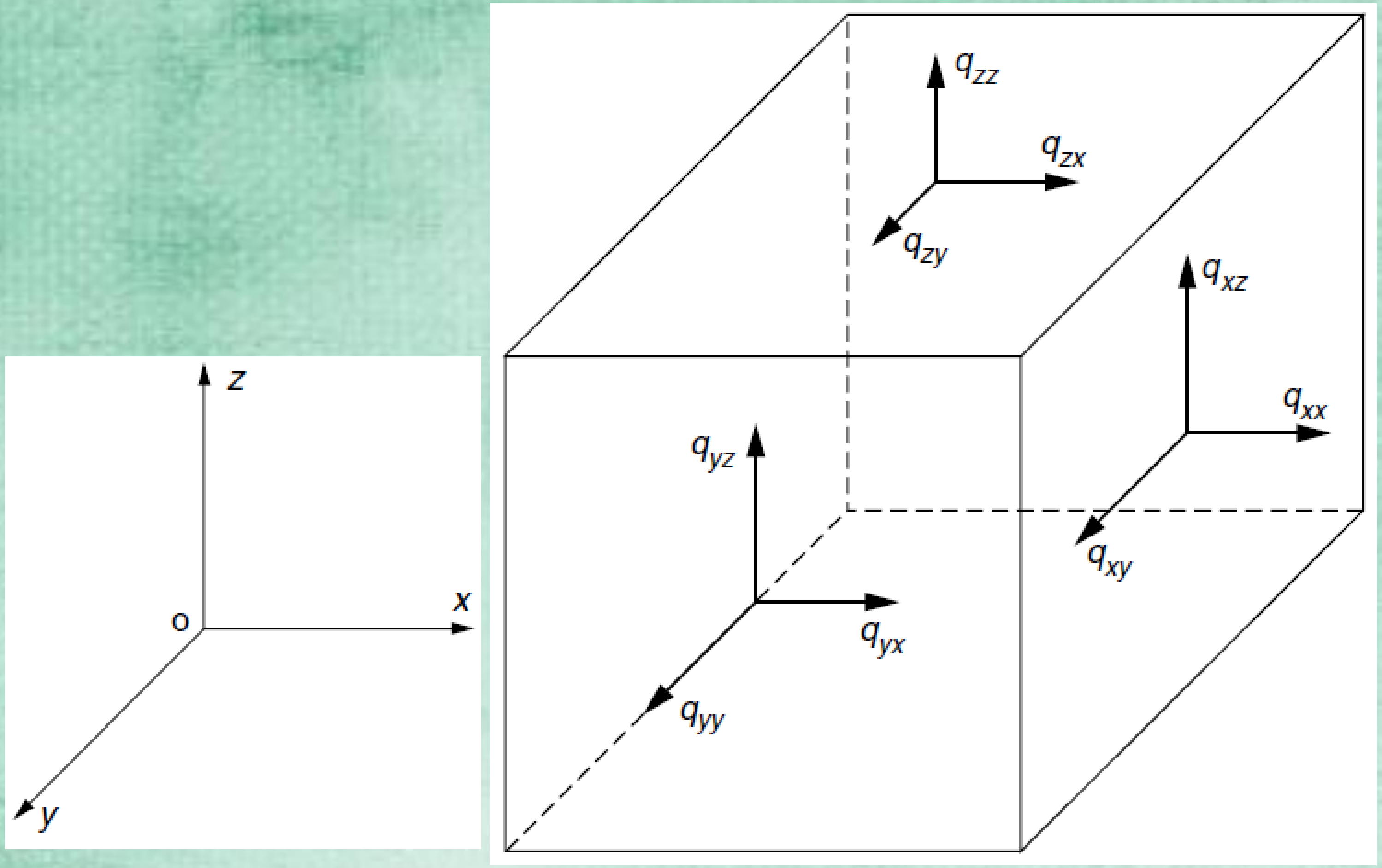
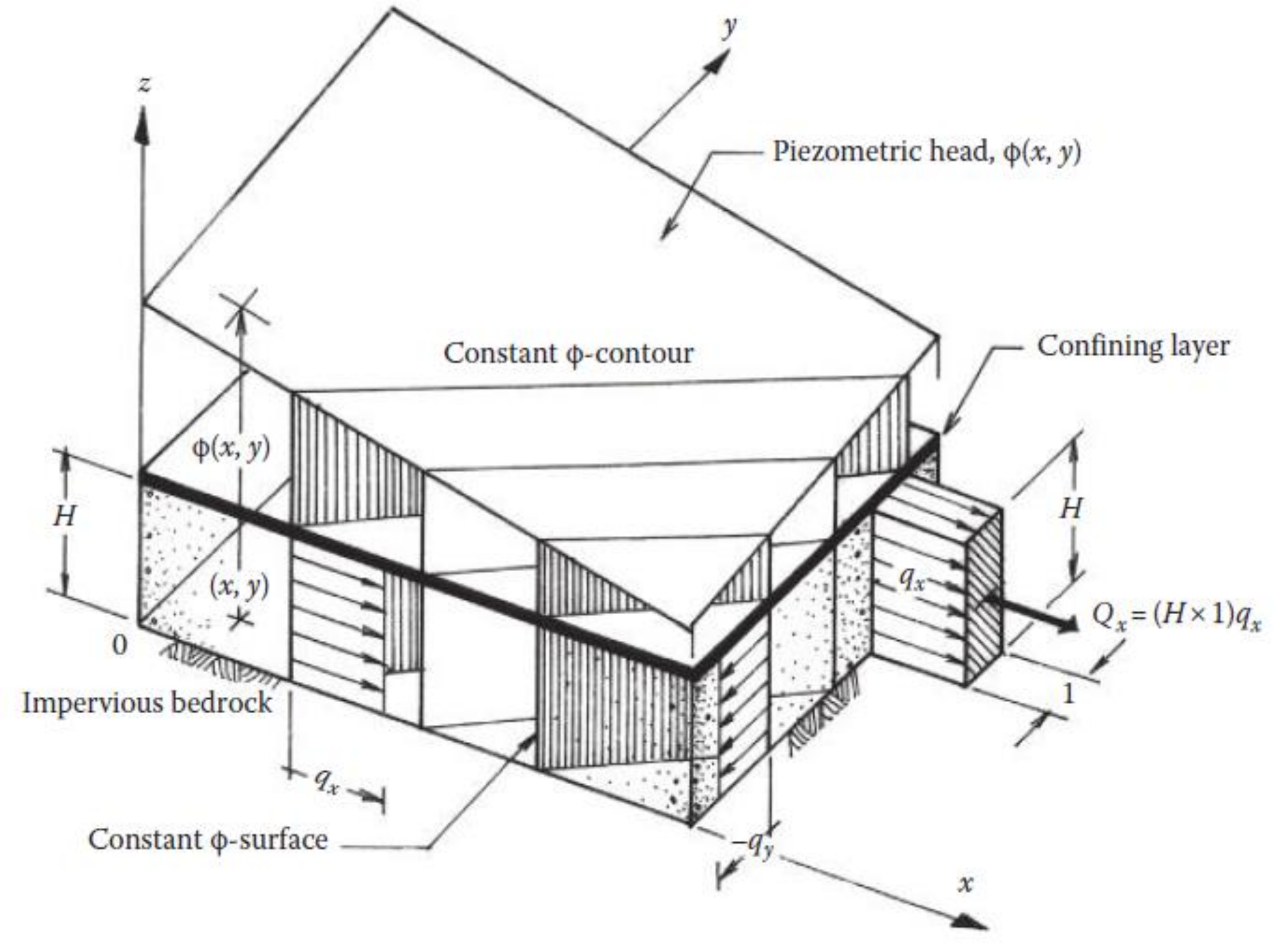


FIGURE 4-15 Darcy velocity components for the general case.

(Ref.: Applied Flow and Solute Transport Modeling in Aquifers, Batu, 2006)

معادله جریان در آبخوانهای تحت فشار



الگوی جریان دوبعدی در یک آبخوان تحت فشار

معادله جریان در آبخوانهای تحت فشار

The governing equation for flow through a porous medium is traditionally derived by referring to the flux of water through a cube of porous material that is large enough to be representative of the properties of the porous medium yet small enough so that the change of head within the volume is relatively small (Fig. 3.1). This cube of porous material is known as a *representative elementary volume or REV*, with volume equal to $\Delta x \Delta y \Delta z$.

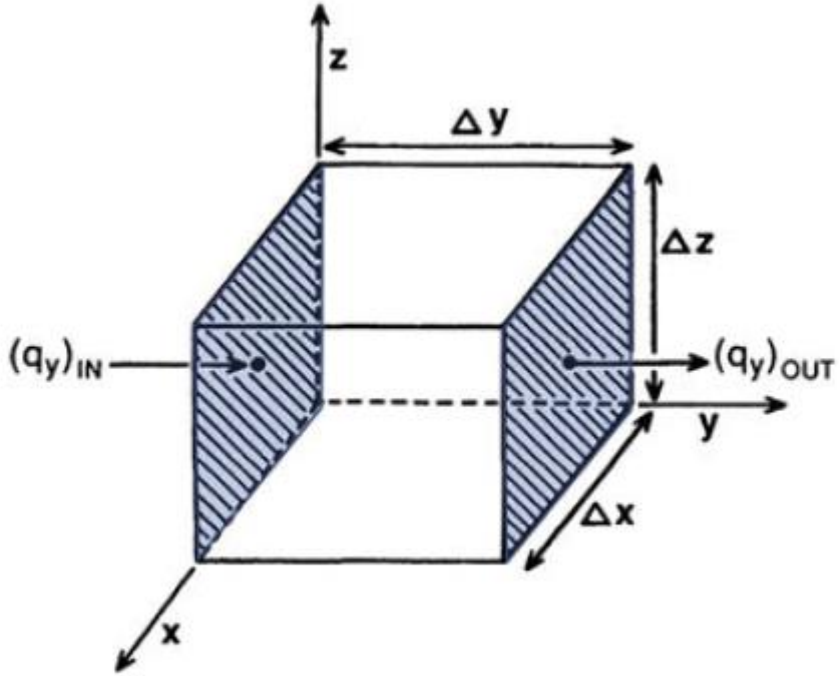


Figure 3.1 Representative elementary volume ($\Delta x \Delta y \Delta z$) showing the components of flow along the *y*-coordinate axis.

معادله جریان در آبخوانهای تحت فشار

The flux through the REV, $\mathbf{q}$, is a vector whose magnitude is expressed by three components, q_x, q_y, and q_z. Formally, we write:

$$\mathbf{q} = q_x \mathbf{i_x} + q_y \mathbf{i_y} + q_z \mathbf{i_z} \tag{3.1}$$

where $\mathbf{i}_x$, $\mathbf{i}_y$, and $\mathbf{i}_z$ are unit vectors along the x, y, and z axes, respectively. Conservation of mass requires a water balance within the REV such that,

$$\text{outflow} - \text{inflow} = \Delta\ \text{storage} \tag{3.2}$$

Consider flow along the y-axis of the REV in Fig. 3.1. Inflow occurs through the face $\Delta x \Delta z$ and is equal to $(q_y)_{IN}$. Outflow is equal to $(q_y)_{OUT}$. The volumetric outflow rate minus the volumetric inflow rate along the y-axis is:

$$\left[(q_y)_{OUT} - (q_y)_{IN}\right]\Delta x \Delta z \tag{3.3}$$

which can be written as,

$$\frac{(q_y)_{OUT} - (q_y)_{IN}}{\Delta y}(\Delta x \Delta y \Delta z) \tag{3.4}$$

Dropping the IN and OUT subscripts and converting from difference notation to a derivative, the change in flow rate through the REV along the y-axis is:

$$\frac{\partial q_y}{\partial y}(\Delta x \Delta y \Delta z) \tag{3.5}$$

معادله جریان در آبخوانهای تحت فشار

Similar expressions are written for the change in flow rate along the x- and z-axes. Using Eqn (3.2), the total change in flow rate is equal to the change in storage:

$$\left(\frac{\partial q_x}{\partial x} + \frac{\partial q_y}{\partial y} + \frac{\partial q_z}{\partial z}\right)\Delta x \Delta y \Delta z = \Delta \text{storage} \tag{3.6}$$

We must allow for the possibility of a sink (e.g., a pumping well) or source of water (e.g., an injection well or recharge) within the REV. The volumetric inflow rate from sources and sinks is represented by $W^* \, \Delta x \Delta y \Delta z$, where we use the convention that W^* is positive when it is a source of water. As a source of water, W^* is subtracted from the left-hand side of Eqn (3.6) (notice the minus sign in front of inflow in Eqn (3.2)), resulting in:

$$\left(\frac{\partial q_x}{\partial x} + \frac{\partial q_y}{\partial y} + \frac{\partial q_z}{\partial z} - W^*\right)\Delta x \Delta y \Delta z = \Delta \text{storage} \tag{3.7}$$

Now consider the right-hand side of Eqn (3.7). Change in storage is represented by specific storage (S_s), which is the volume of water released from storage per unit change in head (h) per unit volume of aquifer:

$$S_s = -\frac{\Delta V}{\Delta h \Delta x \Delta y \Delta z} \tag{3.8}$$

معادله جریان در آبخوانهای تحت فشار

The convention in Eqn (3.8) is that ΔV is intrinsically positive when Δh is negative, or in other words, water is released from storage when head decreases. The rate of change in storage in the REV is:

$$\frac{\Delta V}{\Delta t} = -S_s \frac{\Delta h}{\Delta t} \Delta x \Delta y \Delta z \qquad (3.9)$$

Combining Eqns (3.7) and (3.9) and dividing through by $\Delta x \Delta y \Delta z$ yields the final form of the water balance equation:

$$\frac{\partial q_x}{\partial x} + \frac{\partial q_y}{\partial y} + \frac{\partial q_z}{\partial z} - W^* = -S_s \frac{\partial h}{\partial t} \qquad (3.10)$$

This equation is of little practical use, however, because we cannot easily measure **q**. We want a governing equation written in terms of head because head is an observed quantity easily measured in wells. Darcy's law ($\mathbf{q} = -\underline{\mathbf{K}}\ \mathbf{grad}\ h$) relates specific discharge (**q**) to head (h) where **grad** h is the gradient of h. Both **q** and **grad** h are vectors and $\underline{\mathbf{K}}$ is the hydraulic conductivity tensor (Box 3.1). The components of the specific discharge vector, **q**, are:

$$q_x = -K_x \frac{\partial h}{\partial x}$$
$$q_y = -K_y \frac{\partial h}{\partial y} \qquad (3.11)$$
$$q_z = -K_z \frac{\partial h}{\partial z}$$

where K_x, K_y, and K_z are the principal components of the hydraulic conductivity tensor $\underline{\mathbf{K}}$ and $\partial h/\partial x$, $\partial h/\partial y$, and $\partial h/\partial z$ are components of the vector **grad** h, the gradient of head.

Equation (3.11) is substituted into Eqn (3.10) to give the general governing equation (differential equation) representing three-dimensional (3D) transient groundwater flow for heterogeneous and anisotropic conditions:

$$\frac{\partial}{\partial x}\left(K_x \frac{\partial h}{\partial x}\right) + \frac{\partial}{\partial y}\left(K_y \frac{\partial h}{\partial y}\right) + \frac{\partial}{\partial z}\left(K_z \frac{\partial h}{\partial z}\right) = S_s \frac{\partial h}{\partial t} - W^* \qquad (3.12)$$

The variable of interest, h, is the dependent variable, while x, y, z, and t are the independent variables and K_x, K_y, K_z, S_s, and W^* are parameters. The subscripts on K denote anisotropic conditions (Box 3.1), meaning that hydraulic conductivity can vary with direction, x, y, and z. The placement of K within the differential signs allows for spatial variation (heterogeneity) in hydraulic conductivity.

Equation (3.12) assumes that the principal components of the hydraulic conductivity tensor (K_x, K_y, K_z) are aligned with the coordinate axes x, y, z. When this is not the case, it is necessary to use a version of the governing equation that includes all nine components of the hydraulic conductivity tensor, rather than just the three principal components in Eqn (3.12) (Box 3.1).

Equation (3.12) is used in most numerical groundwater flow codes. The equation simplifies when the problem is steady state ($\partial h/\partial t = 0$) and/or when two-dimensional (2D). For 2D horizontal flow through a confined aquifer, vertically integrated parameters, i.e., transmissivity (T) and storativity (S), can be defined. Then the components of transmissivity in the x- and y-directions are $T_x = K_x b$ and $T_y = K_y b$, respectively, where b is aquifer thickness; $S = S_s b$. The sink/source term, W^*, in Eqn (3.12) becomes a flux, expressed as volume of water per area of aquifer per time, R (L/T). Under these conditions Eqn (3.12) simplifies to: ($R=W^*.b$)

$$\frac{\partial}{\partial x}\left(T_x\frac{\partial h}{\partial x}\right) + \frac{\partial}{\partial y}\left(T_y\frac{\partial h}{\partial y}\right) = S\frac{\partial h}{\partial t} - R \qquad (3.13a)$$

For 2D horizontal flow in an unconfined, heterogeneous, anisotropic aquifer, the differential equation is:

$$\frac{\partial}{\partial x}\left(K_x h\frac{\partial h}{\partial x}\right) + \frac{\partial}{\partial y}\left(K_y h\frac{\partial h}{\partial y}\right) = S_y\frac{\partial h}{\partial t} - R \qquad (3.13b)$$

where S_y is specific yield and R is recharge rate. Here, head (h) is equal to the elevation of the water table measured from the base of the aquifer. 2D horizontal flow, expressed by Eqns 3.13(a) and (b), represents flow under the Dupuit-Forchheimer approximation (Section 4.1; Box 4.1). For steady-state flow with no recharge ($R = 0$) in a homogenous and isotropic aquifer, Eqns 3.13(a) and (b) simplify to the well-known Laplace equation (Section 3.4).

- معادله جریان سه بعدی، آبخوان تحت فشار ناهمگن ناهمسانگرد:

$$\frac{\partial}{\partial x}\left(K_x \frac{\partial h}{\partial x}\right) + \frac{\partial}{\partial y}\left(K_y \frac{\partial h}{\partial y}\right) + \frac{\partial}{\partial z}\left(K_z \frac{\partial h}{\partial z}\right) = S_s \frac{\partial h}{\partial t}$$

- معادله جریان سه بعدی، آبخوان تحت فشار همگن ناهمسانگرد:

$$K_x \frac{\partial^2 h}{\partial x^2} + K_y \frac{\partial^2 h}{\partial y^2} + K_z \frac{\partial^2 h}{\partial z^2} = S_s \frac{\partial h}{\partial t}$$

- معادله جریان سه بعدی، آبخوان تحت فشار همگن همسانگرد:

$$\frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial y^2} + \frac{\partial^2 h}{\partial z^2} = \frac{S_s}{K} \frac{\partial h}{\partial t}$$

- معادله جریان دو بعدی، آبخوان تحت فشار همگن همسانگرد:

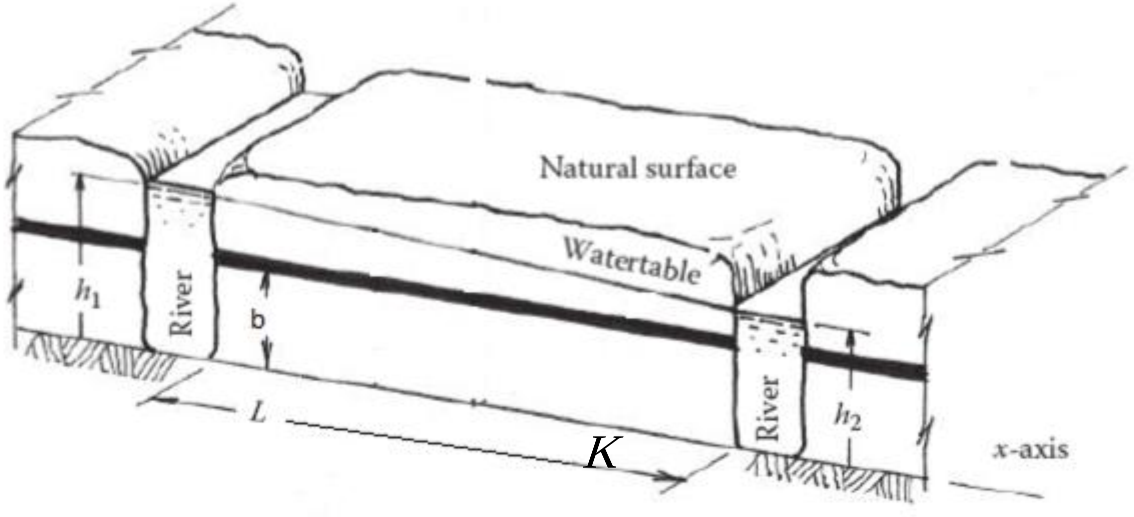
$$\frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial y^2} = \frac{S}{T} \frac{\partial h}{\partial t}$$

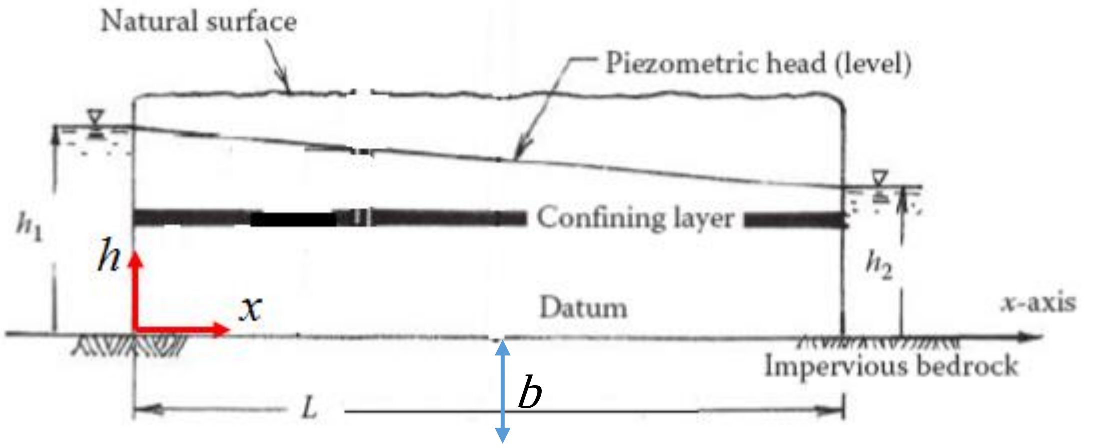
- معادله جریان دو بعدی، آبخوان تحت فشار همگن همسانگرد:

$$\frac{\partial^2 s}{\partial x^2} + \frac{\partial^2 s}{\partial y^2} = \frac{S}{T} \frac{\partial s}{\partial t}$$

- معادله جریان سه بعدی، آبخوان تحت فشار همگن همسانگرد، رژیم ماندگار (معادله لاپلاس):

$$\frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial y^2} + \frac{\partial^2 h}{\partial z^2} = 0 \qquad or \quad \nabla^2 h = 0$$





شکل ۴-۲- جریان یک‌بعدی در آبخوان تحت فشار

- معادله جریان یک بعدی، آبخوان تحت فشار همگن همسانگرد:

$$\frac{d^2h}{dx^2}=0 \quad \Rightarrow \frac{dh}{dx}=C_1 \quad \Rightarrow h(x)=C_1x+C_2$$

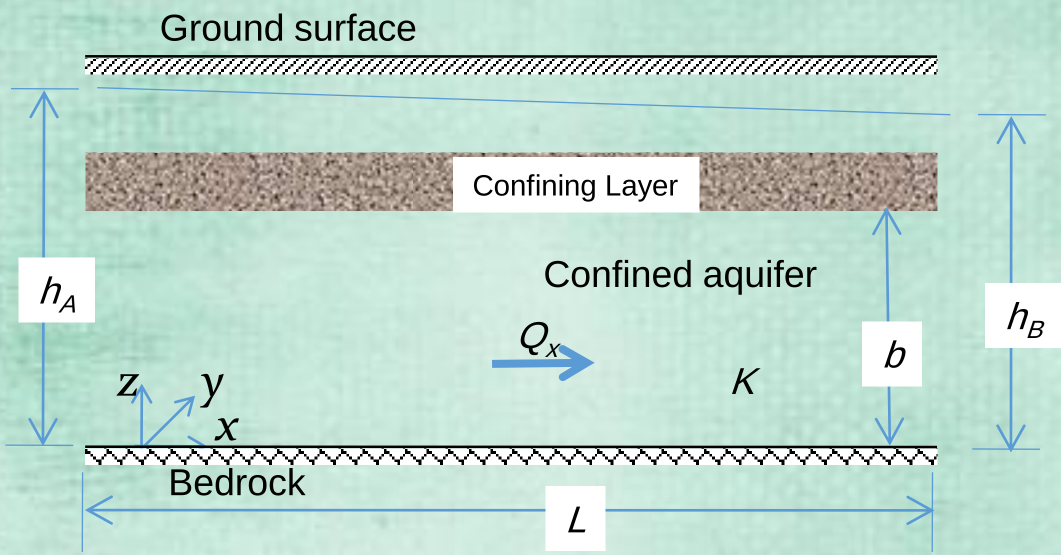
از شکل ۴-۱ در $x=0$ داریم $h=h(0)=h_1$ و در $x=L$ داریم $h=h(L)=h_2$ و با قرار دادن در معادله بالا:

$C_2=h_1,\ C_1=(h_1-h_2)/L$

$$\Rightarrow \quad h(x)=h_1+\frac{h_2-h_1}{L}x \quad or \quad h(x)=h_1-ix$$

در رابطه بالا i قدرمطلق گرادیان هیدرولیکی (چون مقدار آن منفی است).

نتیجه مهم این است که در این جریان اولاً تغییرات هد خطی است و ثانیاً هد مستقل از هدایت هیدرولیکی است. مثال:



L = 1000 m, h_A = 100 m, h_B = 80 m,

K = 20 m/d, ϕ = 0.35

$$h(x)=h_A+\frac{h_B-h_A}{L}x=100-0.02x \quad m$$

$$q=-K\frac{h_B-h_A}{L}=-(20)\frac{80-100}{1000}=0.4\,m/day$$

$$v_s=\frac{q}{\phi}=1.14\,m/day$$

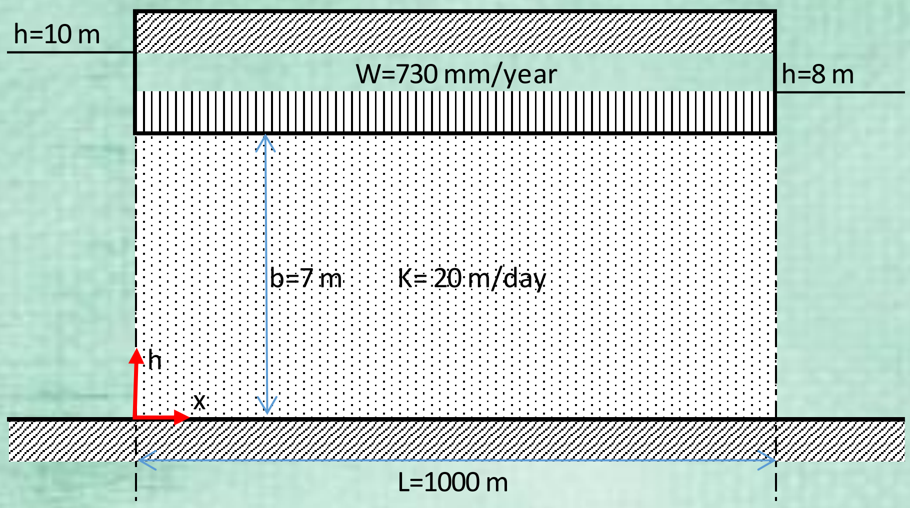
جریان یک بعدی ماندگار با نشت عمودی

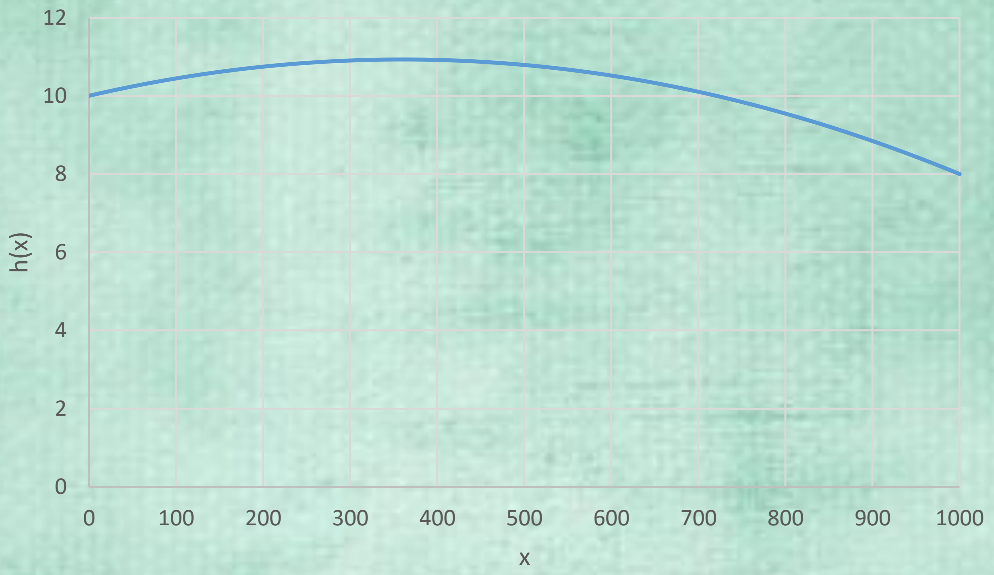
$$\frac{d^2h}{dx^2}+\frac{W}{T}=0 \quad \Rightarrow \frac{dh}{dx}=-\frac{W}{T}x+C_1 \quad \Rightarrow h(x)=-\frac{W}{2T}x^2+C_1x+C_2$$

$$h(0)=h_1; \quad h(L)=h_2 \Rightarrow$$

$$h(x)=-\frac{W}{2T}x^2+\left(\frac{h_2-h_1}{L}+\frac{WL}{2T}\right)x+h_1$$

مثال:





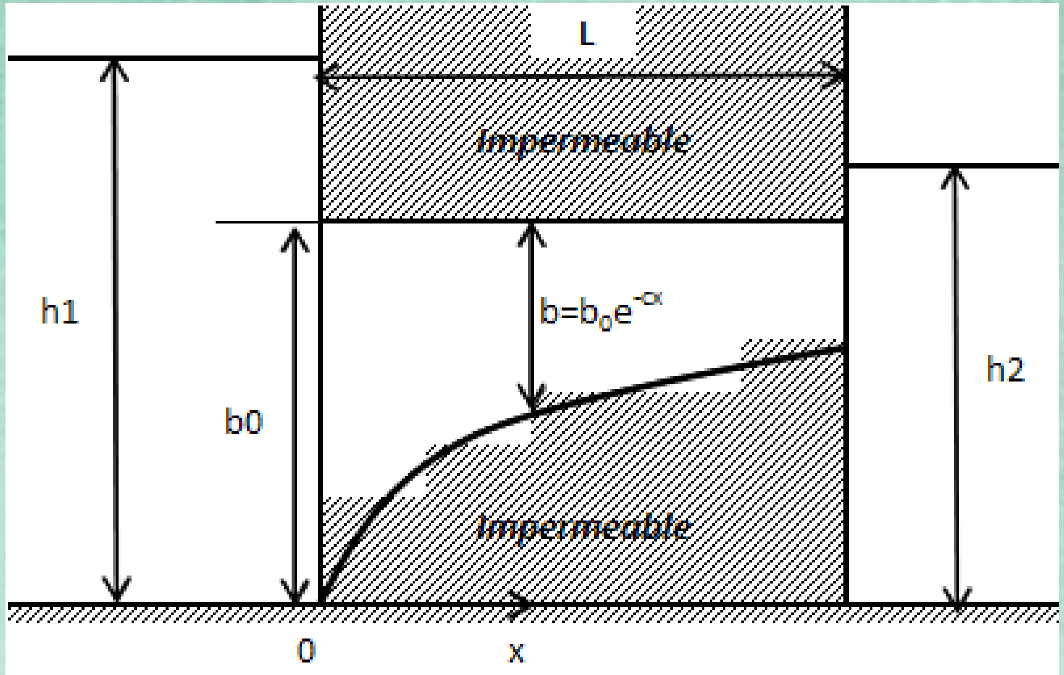
$$x_{\max}=\frac{L}{2}+\frac{T}{W}\left(\frac{h_2-h_1}{L}\right)$$

Example – Varying Thickness

Determine the hydraulic head distribution in the confined aquifer below. Aquifer is homogeneous isotropic and steady flow conditions. (after Hantush, 1962).

$$\frac{\partial}{\partial x}(T_x \frac{\partial h}{\partial x}) = 0;$$

$$\frac{\partial}{\partial x}(K_x . b_x \frac{\partial h}{\partial x}) = 0;$$



$$h(0) = h_1, \quad h(L) = h_2; \quad and$$

$$b = b_0 e^{-cx} \quad \Rightarrow$$

$$b\frac{d^2h}{dx^2} + \frac{db}{dx}\frac{dh}{dx} = 0 \quad \Rightarrow$$

$$\frac{d^2h}{dx^2} - c\frac{dh}{dx} = 0 \quad \Rightarrow h = C_1 e^{cx} + C_2 \quad \Rightarrow h = h_1 - \left[\frac{(h_1 - h_2)(e^{cx} - 1)}{(e^{cL} - 1)}\right]$$

جریان یک بعدی در آبخوان تحت فشار

معادله لاپلاس در جریان یک بعدی در جهت X عبارتست از:

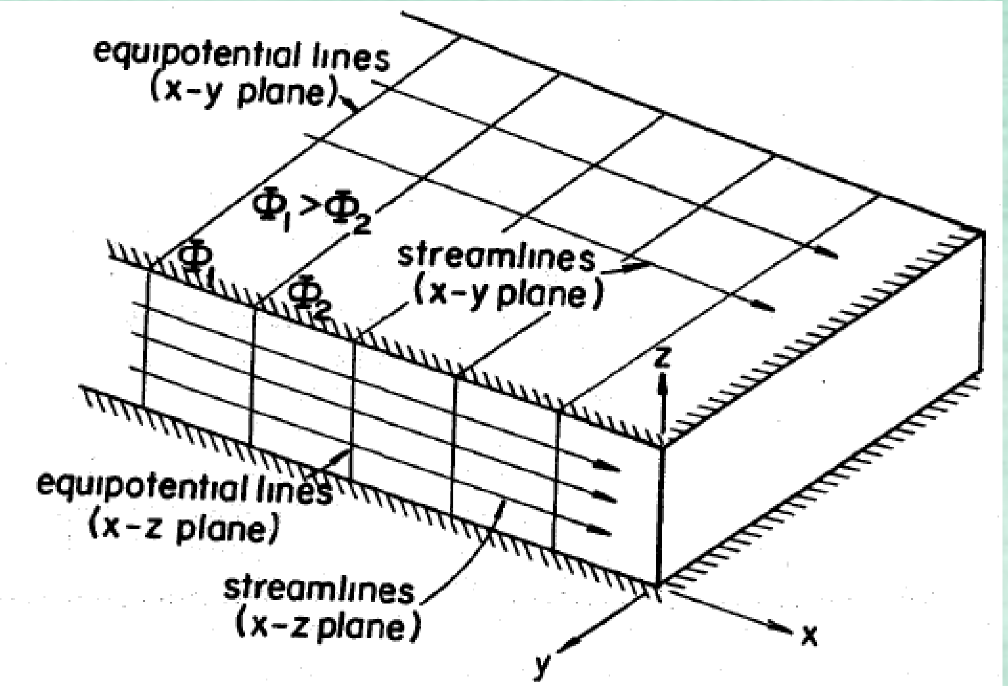
$$\frac{d^2\Phi}{dx^2} = 0$$

با دوبار انتگرال‌گیری و برابر قرار دادن ثابت ضریب X برابر $-q_X$ و ثابت دوم برابر صفر داریم:

$$\Phi = -q_x x$$

این جریان یک بعدی در شکل ۴-۳ نشان داده شده است. همچنین معادله جریان در این حالت عبارتست از:

$$\psi = -\int q_x dz + f(x) = -q_x z$$



The streamlines corresponding to $\psi=\psi_i$ are horizontal lines, parallel to the x-axis. It is clear that the streamline $\psi=0$ is the lower boundary of the aquifer, and any other value, ψ_i at z_i, represents the negative discharge rate between the streamlines $\psi=\psi_i$ and $\psi=0$, per unit of aquifer width measured along y.

شکل ۴-۳- جریان یک بعدی
یکنواخت در یک
آبخوان تحت فشار

معادله جریان در آبخوان تحت فشار نشتی با ورودی عمودی

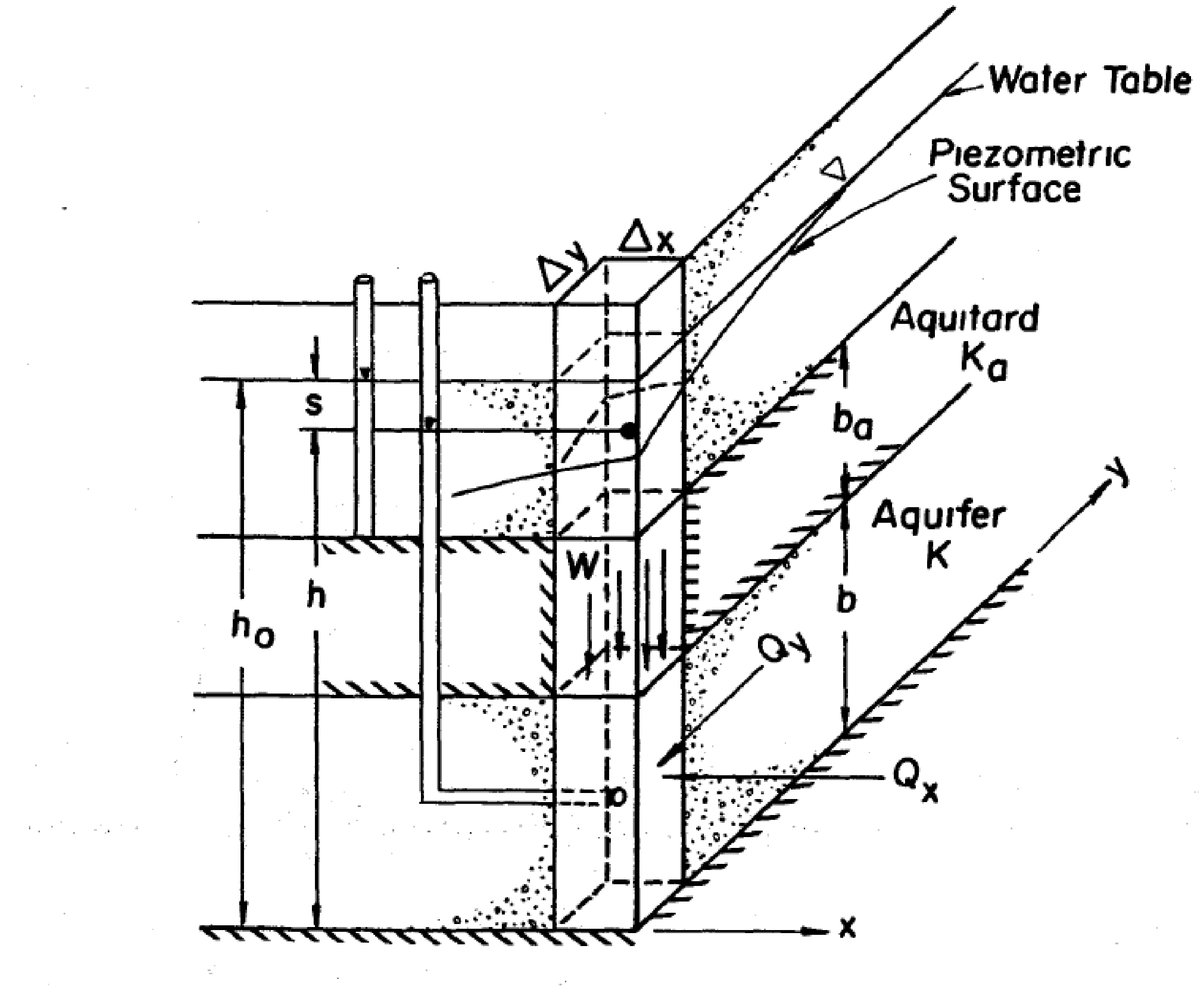


Figure 3-16. Flow in a leaky, confined aquifer.

$$\frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial y^2} + \frac{W}{bK} = \frac{S}{bK}\frac{\partial h}{\partial t} \quad , \qquad (3\text{-}76)$$

W=W*.b=R

The vertical percolation rate W is a scalar discharge per unit area assumed to be positive under conditions of accretion. The magnitude of W can be computed directly from Darcy's Law, provided that changes in storage in the aquitard are neglected (see Examples 3-6 and 3-7):

$$W = K_a \frac{(h_o - h)}{b_a} \quad . \qquad (3\text{-}78)$$

Substitution of Eq. 3-78 into Eq. 3-76 gives

$$\frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial y^2} + \frac{K_a}{K}\frac{(h_o - h)}{bb_a} = \frac{S}{bK}\frac{\partial h}{\partial t} \quad . \qquad (3\text{-}79)$$

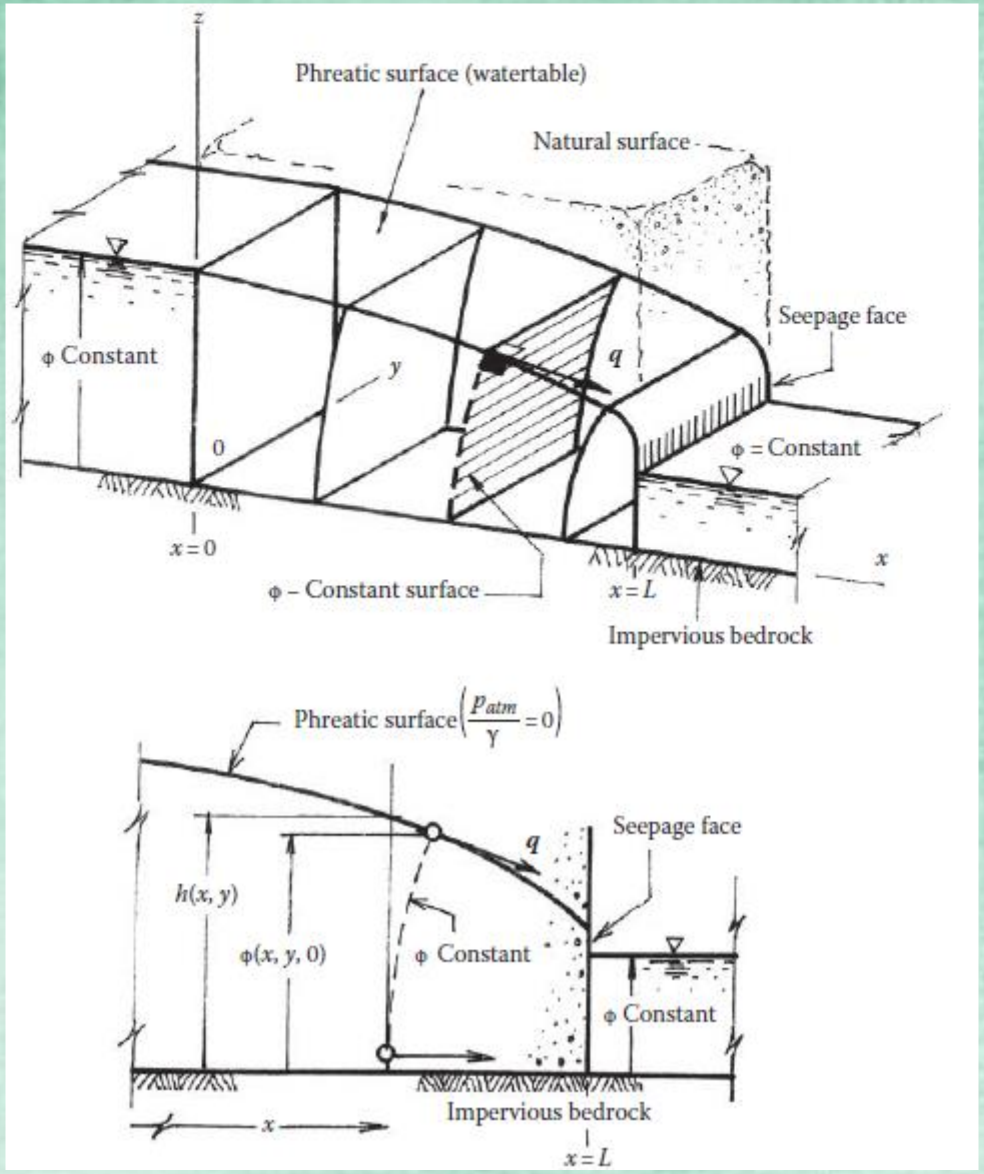
A *leakage factor B*, defined by

$$B = \left(\frac{Kbb_a}{K_a} \right)^{1/2} , \qquad (3\text{-}80)$$

is often introduced into Eq. 3-79 to yield the *leaky* aquifer equation

$$\frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial y^2} + \frac{h_o - h}{B^2} = \frac{S}{bK} \frac{\partial h}{\partial t} . \qquad (3\text{-}81)$$

معادله جریان در آبخوان های آزاد



شکل ۳–۱۵–الگوی جریان در یک آبخوان آزاد

معادله جریان در آبخوان های آزاد

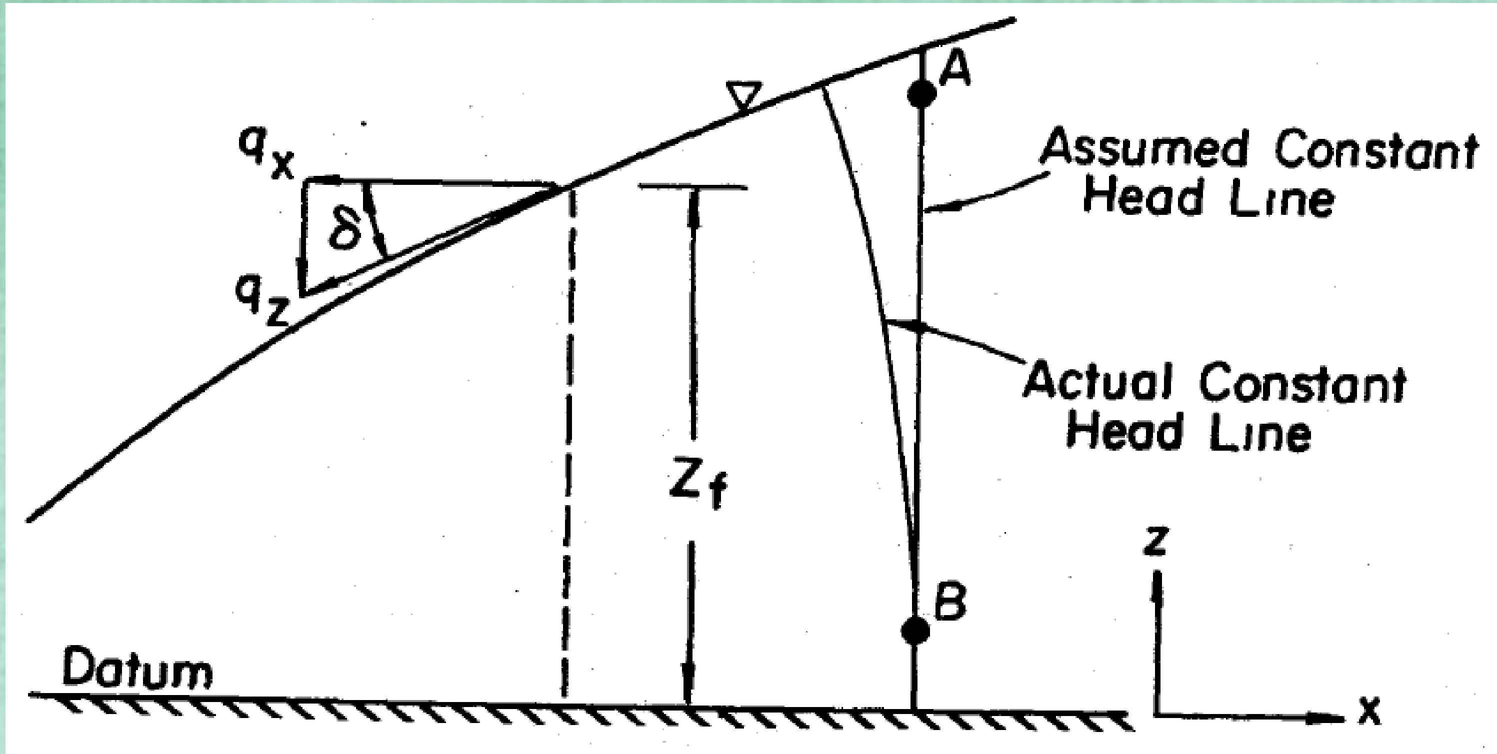
The Dupuit-Forchheimer Approximations

The difficulties attending the solution of Eq. 3-64, or appropriate less general forms, in water-table aquifers have led hydrologists to use a more practical, if less rigorous, approach. Consider a sloping water table above a horizontal impermeable boundary as shown in Fig. 3-14. The slope has been greatly exaggerated for clarity. The discharge, per unit width into the plane of the paper, across any vertical plane is

$$Q = \int_0^{z_f} q_x(x,z)\, dz \quad . \qquad (3\text{-}65)$$

Evaluation of the integral in Eq. 3-65 requires that $q_x(x,z)$ be known. However, provided that the slope δ of the water table is small, q_x at the water table does not differ significantly from that on the impermeable boundary and $q_x(x,z) \approx q_x(x,z_f)$.

معادله جریان در آبخوان های آزاد



شکل ۳-۱۹-فرض دوپویی در آبخوان آزاد

معادله جریان در آبخوان های آزاد

In this case

$$Q = q_x(x,z_f)\ z_f = -K \frac{dh}{dx} z_f \quad , \qquad (3\text{-}66)$$

where h is the piezometric head at the water table. By definition of a water table, the pressure head must be zero there,

so $h=z_f$ and

$$Q = -Kh \frac{dh}{dx} \quad . \qquad (3\text{-}67)$$

In Eq. 3-67, h represents both the thickness of the flow and the piezometric head at the water table. The quantity dh/dx is the tangent of the angle the water table makes with the horizontal. Equation 3-67 actually implies that the flow is entirely horizontal, and that the pressure-head distribution along any vertical is hydrostatic. In other words, the piezometric head along any vertical is constant. It is emphasized that Eq. 3-67 is valid for situations in which the water-table slope is small. More explicitly

$$(dh/dx)^2 << 1 \qquad (3\text{-}68)$$

is the condition that must be satisfied (Bear, 1972)

The Dupuit-Forchhiemer assumption of horizontal flow permits the use of a material-balance control volume that extends from the horizontal floor of the aquifer to the water table (Fig. 3-15). Because changes in water density are unimportant in unconfined aquifers, mass balance is assured by a volume balance. Following the procedures of the previous section, the rate of net outflow from the control volume is

$$\text{Net Outflow Rate} = \frac{\partial Q_x}{\partial x}\,\Delta x + \frac{\partial Q_y}{\partial y}\,\Delta y \quad . \qquad (3\text{-}69)$$

From Eq. 3-67, written for both the x and y directions in an isotropic aquifer,

$$\frac{\text{Net Outflow Rate}}{\Delta x \Delta y} = -\frac{\partial}{\partial x}\left(Kh\,\frac{\partial h}{\partial x}\right) - \frac{\partial}{\partial y}\left(Kh\,\frac{\partial h}{\partial y}\right) . \qquad (3\text{-}70)$$

As before, the net rate of outflow must equal the negative time rate of reduction of stored water volume. The change in water volume associated with a change, dh, of water-table level follows from the definition of apparent specific yield discussed in Chapter II.

$$\frac{\partial V_w}{\partial t} = S_{ya}\,\frac{\partial h}{\partial t}\,\Delta x \Delta y \quad . \qquad (3\text{-}71)$$

Combining Eqs. 3-70 and 3-71 yields

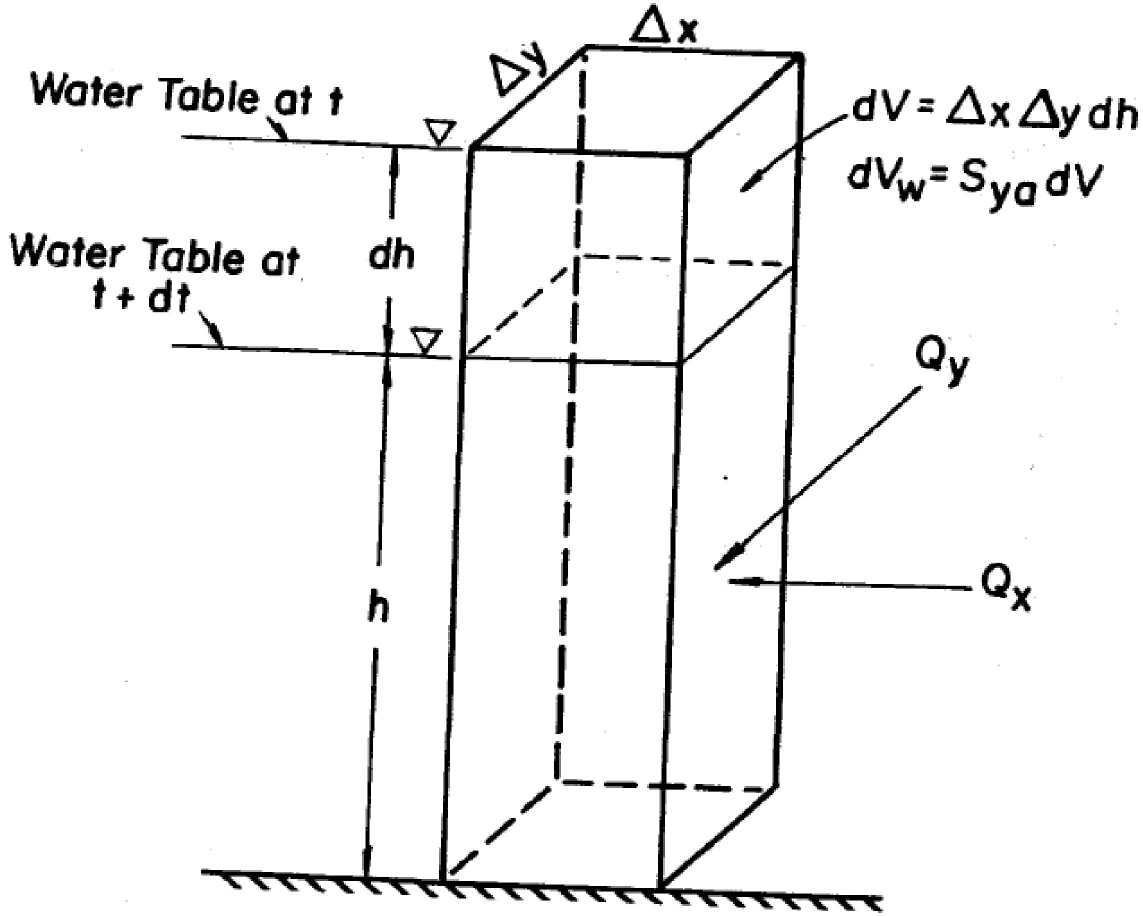


Figure 3-15. Control volume in an unconfined aquifer.

$$\frac{\partial}{\partial x}\left(Kh\frac{\partial h}{\partial x}\right)+\frac{\partial}{\partial y}\left(Kh\frac{\partial h}{\partial y}\right)=S_{ya}\frac{\partial h}{\partial t}\,, \qquad (3\text{-}72)$$

and, if the aquifer is homogeneous,

$$\frac{\partial}{\partial x}\left(h\frac{\partial h}{\partial x}\right)+\frac{\partial}{\partial y}\left(h\frac{\partial h}{\partial y}\right)=\frac{S_{ya}}{K}\frac{\partial h}{\partial t}\,. \qquad (3\text{-}73)$$

معادله جریان در آبخوان های آزاد

- آبخوان ناهمگن ناهمسانگرد

$$\frac{\partial}{\partial x}(K_x h\frac{\partial h}{\partial x})+\frac{\partial}{\partial y}(K_y h\frac{\partial h}{\partial y})+\frac{\partial}{\partial z}(K_z h\frac{\partial h}{\partial z})+W=S_y\frac{\partial h}{\partial t}$$

- آبخوان همگن ناهمسانگرد

$$K_x(\frac{\partial^2 h^2}{\partial x^2})+K_y(\frac{\partial^2 h^2}{\partial y^2})+K_z(\frac{\partial^2 h^2}{\partial z^2})+2W=2S_y\frac{\partial h}{\partial t}$$

- آبخوان همگن همسانگرد

$$\frac{\partial^2 h}{\partial x^2}+\frac{\partial^2 h}{\partial y^2}+\frac{\partial^2 h}{\partial z^2}+\frac{2W}{K}=\frac{2S_y}{K}\frac{\partial h}{\partial t}$$

- آبخوان همگن همسانگرد- جریان ماندگار:

$$\frac{\partial^2 h^2}{\partial x^2}+\frac{\partial^2 h^2}{\partial y^2}+\frac{\partial^2 h^2}{\partial z^2}=0$$

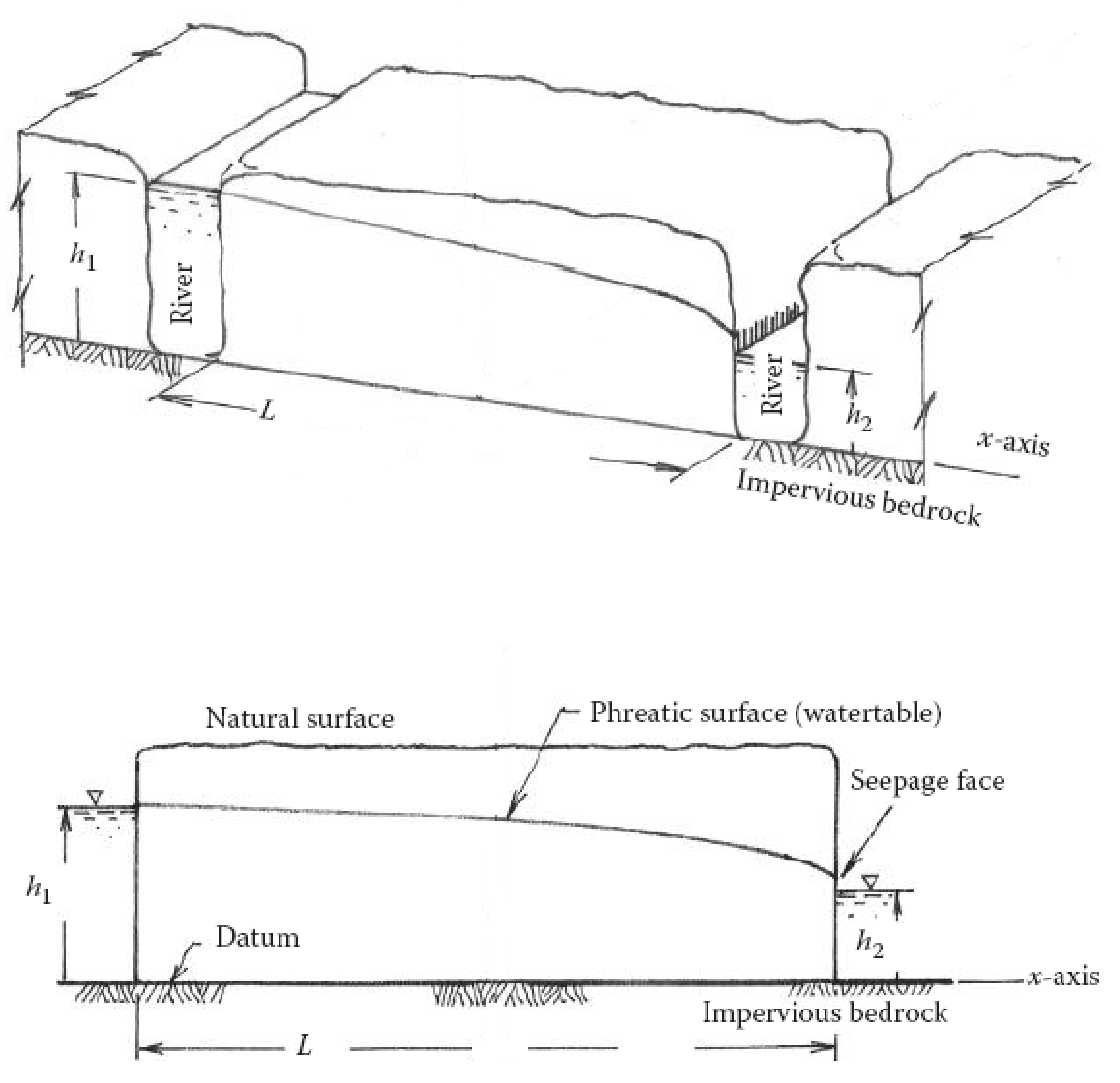
- آبخوان همگن - جریان ماندگار یک بعدی:

$$\frac{\partial^2 h^2}{\partial x^2}=0$$

- آبخوان همگن - جریان ماندگار یک بعدی با نفوذ عمقی:

$$\frac{\partial^2 h^2}{\partial x^2}+\frac{2W}{K}=0$$

جریان ماندگار یک بعدی در آبخوان آزاد



شکل ۴-۲- جریان ماندگار یک‌بعدی در آبخوان آزاد

جریان ماندگار یک بعدی در آبخوان آزاد

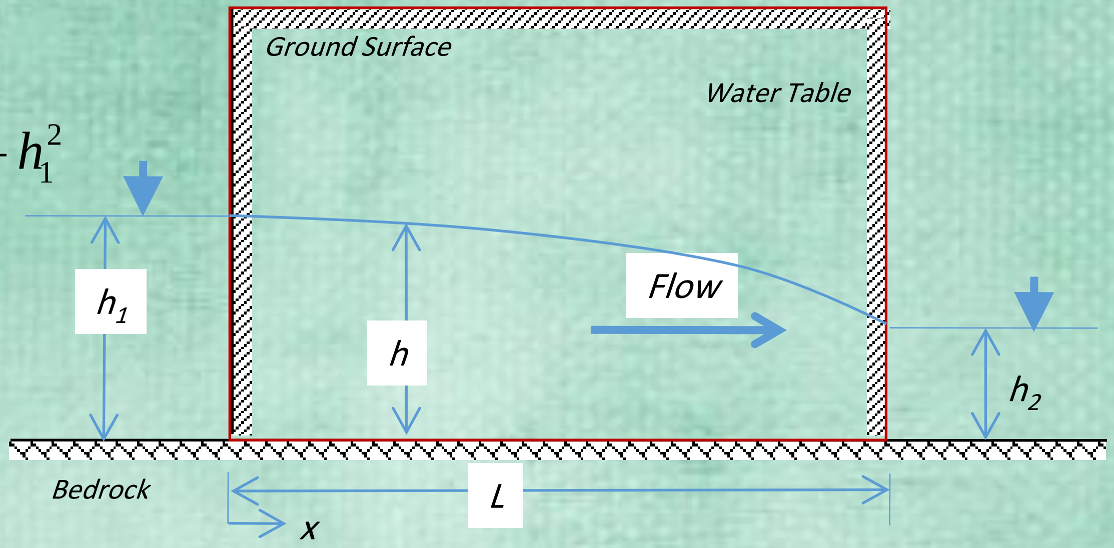
$$\frac{d^2h^2}{dx^2}=0 \Rightarrow \frac{dh^2}{dx}=C_1 \Rightarrow h^2(x)=C_1x+C_2$$

$$in\, x=0, h=h_1 \Rightarrow h^2(0)=h_1^2=C_2;$$

$$in\, x=L, h=h_2 \Rightarrow h^2(L)=h_2^2=C_1L+h_1^2$$

$$\Rightarrow C_1=\frac{h_2^2-h_1^2}{L} \Rightarrow$$

$$h^2(x)=h_1^2+(\frac{h_2^2-h_1^2}{L})x$$

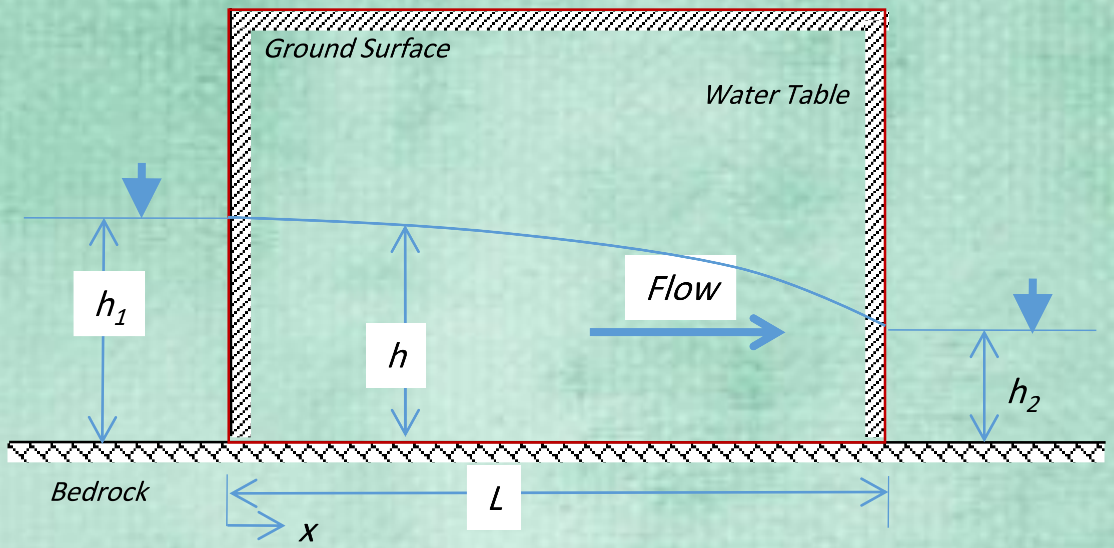


$$Q=(-K\frac{dh}{dx})h=-\frac{K}{2}\frac{dh^2}{dx}=-\frac{K}{2}\left(\frac{h_1^2-h_2^2}{L}\right)$$

جریان ماندگار یک بعدی در آبخوان آزاد

Example:

- $K = 10^{-1}$ cm/sec
- $h_1 = 6.5$ m
- $h_2 = 4$ m
- $L = 150$ m
- Find Q



$$Q = -\frac{K}{2}\left(\frac{h_2^2 - h_1^2}{L}\right) = -\frac{86.4\, m/d}{2}\left(\frac{6.5^2 - 4^2}{150}\right) = 7.56\, m^3/d/m$$

جریان ماندگار یک بعدی در آبخوان آزاد

جریان یک بعدی با نشت عمودی

$$\frac{\partial}{\partial x}\left(Kh\frac{\partial h}{\partial x}\right)+W=0$$

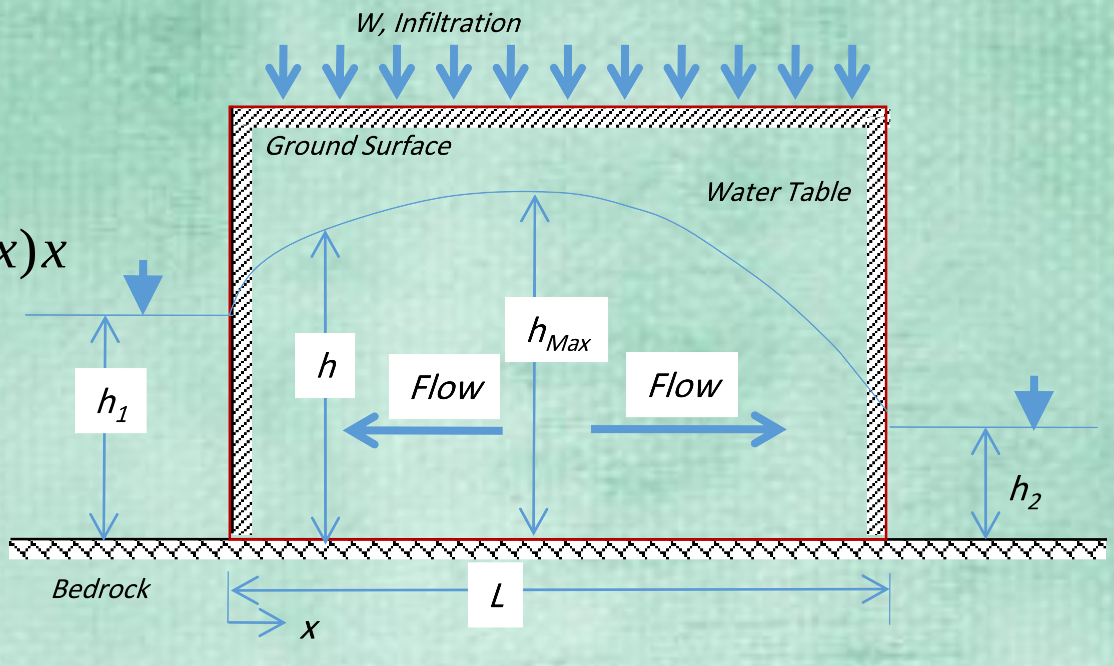
$$\frac{d^2\left(h^2\right)}{dx^2}=-2\frac{W}{K}$$

$$h^2(x)=h_1^2+\left(\frac{h_2^2-h_1^2}{L}\right)x+\frac{W}{K}(L-x)x$$

$$Q(x)=-\frac{K}{2}\frac{d(h^2)}{dx}$$

$$\frac{dh^2}{dx}=\left(\frac{h_2^2-h_1^2}{L}\right)+\frac{W}{K}(L-2x)$$

$$Q(x)=-K\frac{h_2^2-h_1^2}{2L}-W(\frac{L}{2}-x)$$



$$x_{divide}=\frac{L}{2}+\frac{K}{2WL}(h_2^2-h_1^2)$$

جریان ماندگار یک بعدی در آبخوان آزاد

جریان یک بعدی با نشت عمودی

Example

- Given

  L = 3000 m

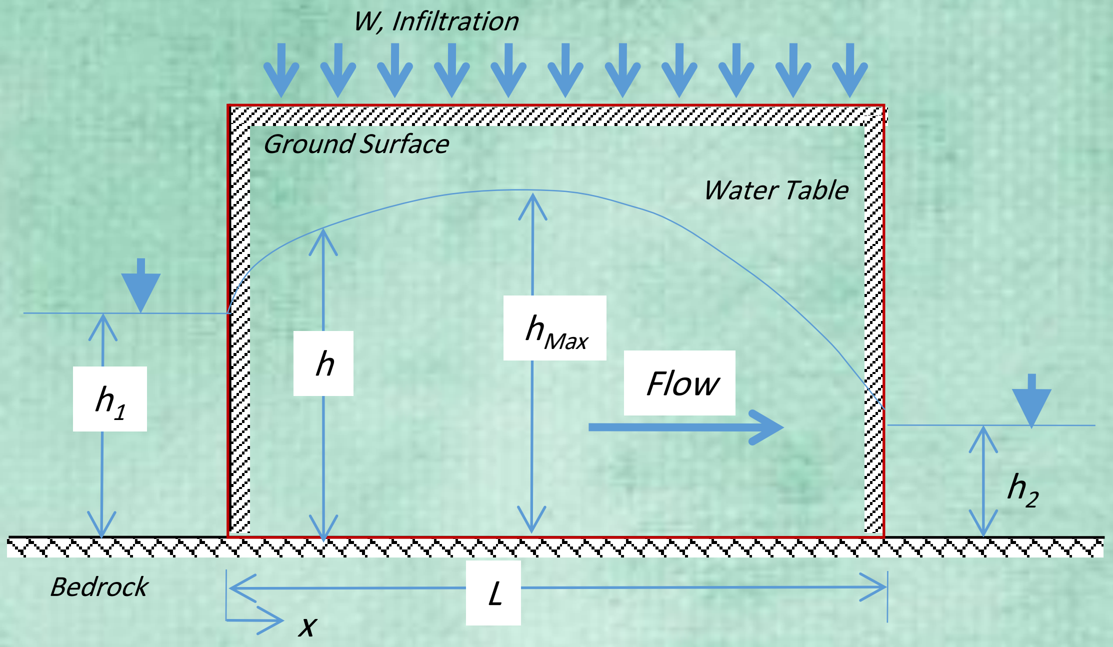
  K = 20 m/day

  h_1 = 30 m

  h_2 = 20 m

  W = 500 mm/yr

  W = 0.00137 m/day



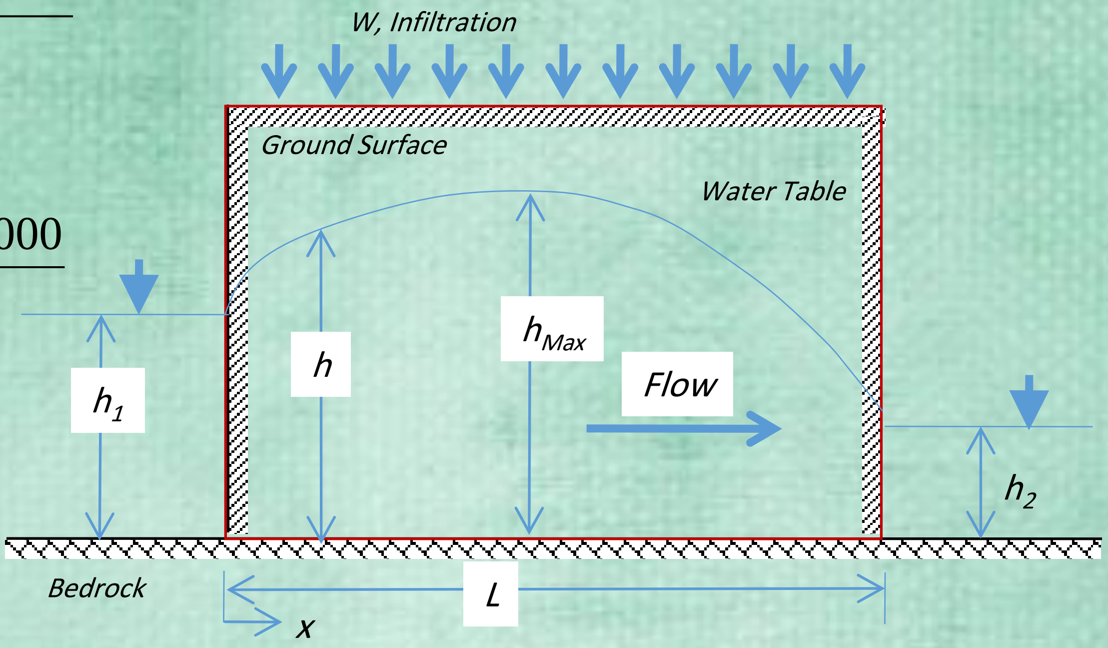
- Find: Flow to the streams and shape of water table

جریان ماندگار یک بعدی در آبخوان آزاد

جریان یک بعدی با نشت عمودی

$$Q(0) = \frac{20}{2\times 3000}(30^2 - 20^2) - \frac{0.00137\times 3000}{2}$$
$$= -0.388\, m^3 / day / m$$

$$Q(L) = \frac{20}{2\times 3000}(30^2 - 20^2) + \frac{0.00137\times 3000}{2}$$
$$= 3.722\, m^3 / day / m$$



$$x_{divide} = \frac{3000}{2} + \frac{20*(-500)}{2*1.37*3}$$
$$= 283.5\, m$$

$$h_{\max} = (30^2 + \left(\frac{20^2 - 30^2}{3000}\right)\times 283.5 + \frac{0.00137}{20}(3000 - 283.5)\times 283.5)^{0.5} = 30.0916$$

جریان ماندگار در آبخوان های آزاد

کاربرد در طراحی زهکش ها

$$\frac{\partial}{\partial x}\left(Kh\frac{\partial h}{\partial x}\right)+W=0.0$$

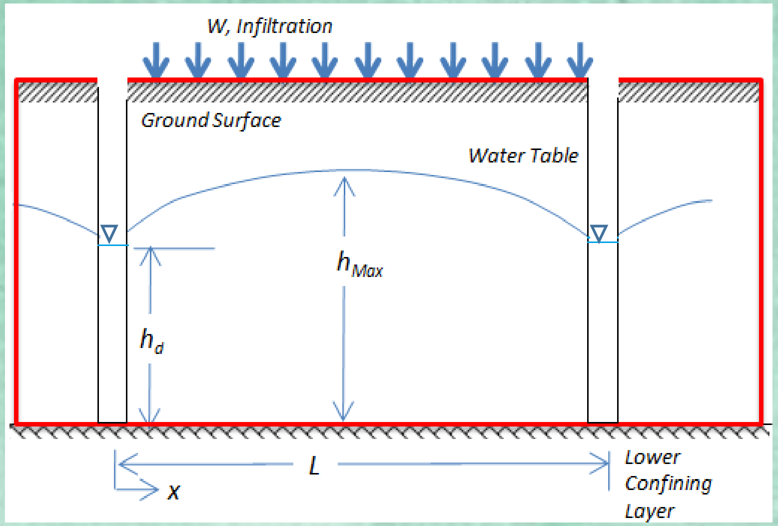
$$\frac{d^2\left(h^2\right)}{dx^2}=-2\frac{W}{K}$$

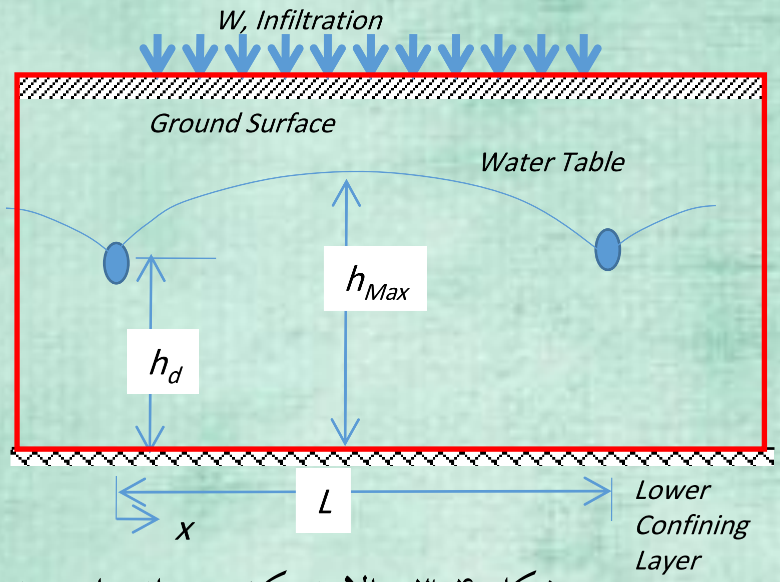
$$h^2=\frac{Wx}{K}(L-x)+h_d^2$$

$$h_{Max}^2=h(0.5L)^2$$

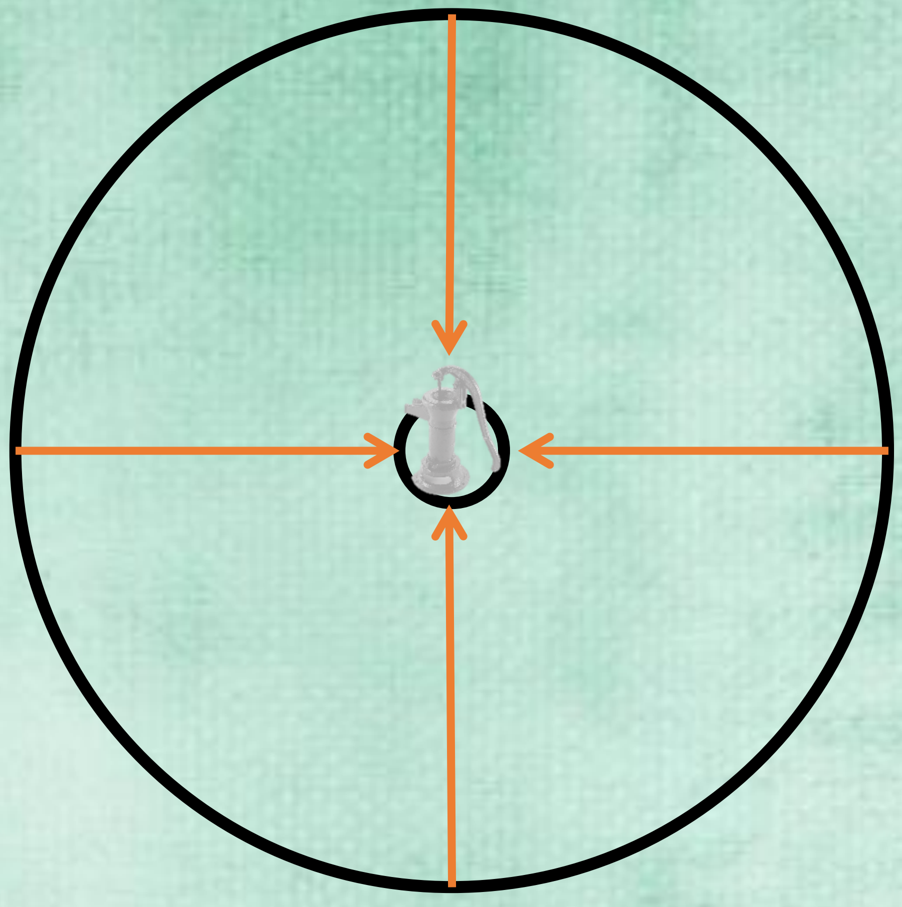
$$=\frac{WL^2}{4K}+h_d^2$$

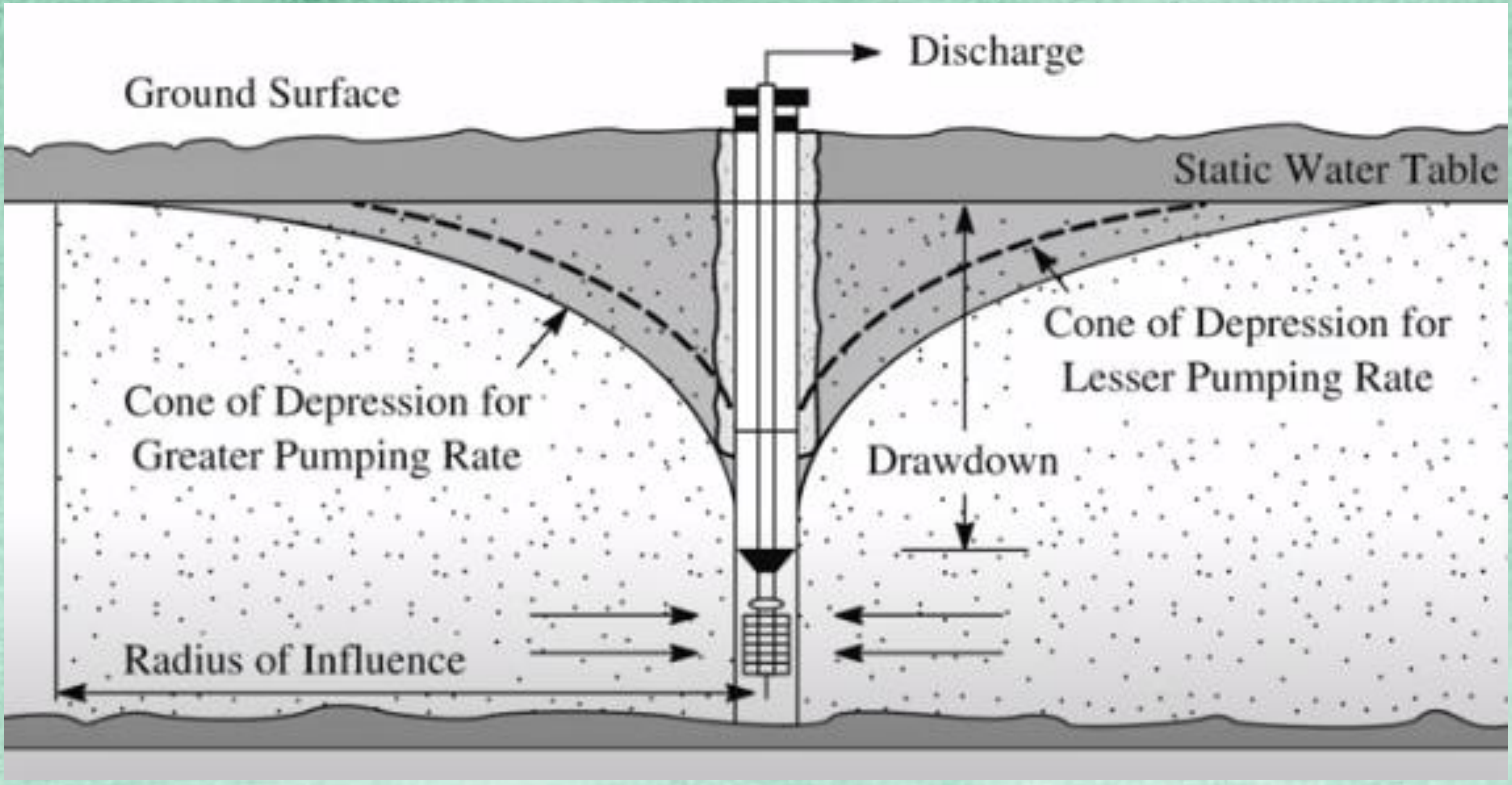
$$Q_0=Q_L=\frac{WL}{2}$$

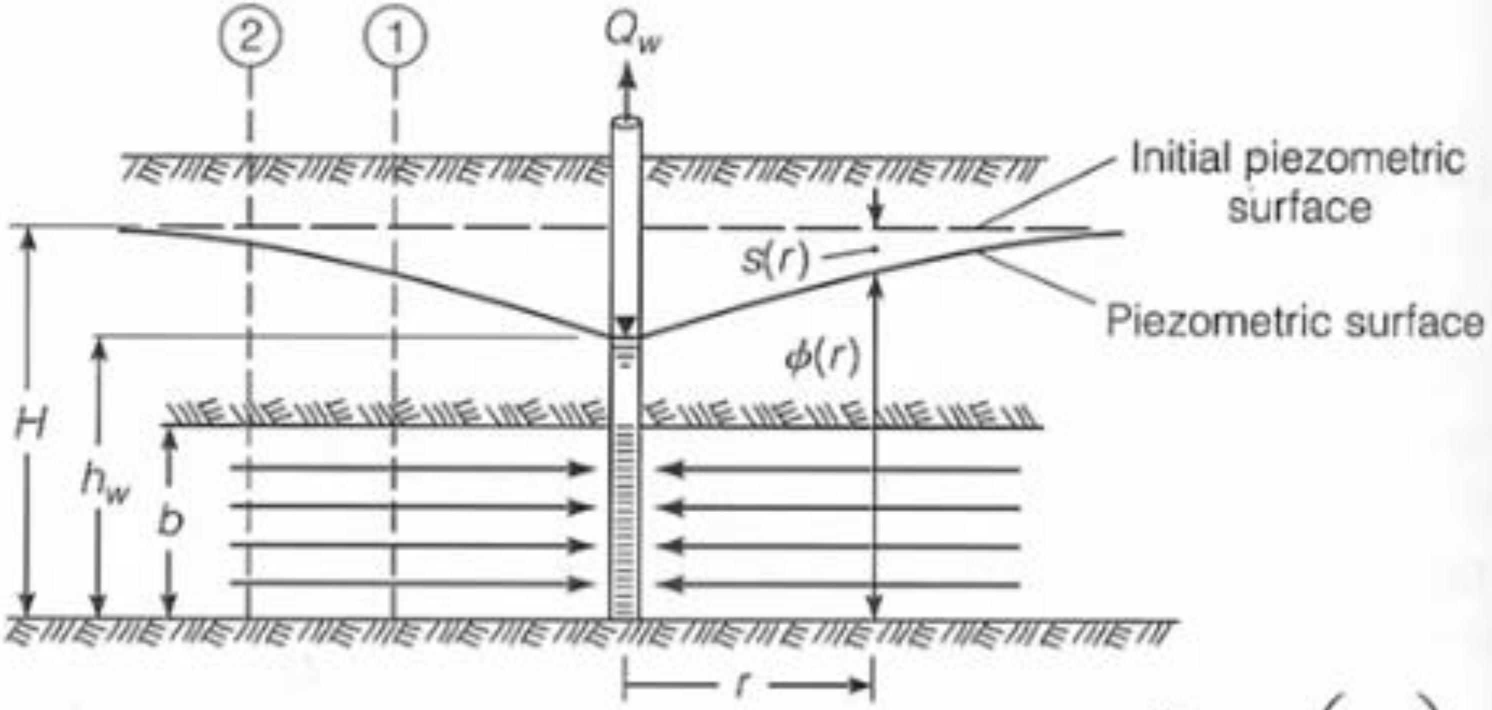




شکل ۴-۳- بالا: زهکش روباز-پایین: زهکش مدفون




Discharge
Ground Surface
Static Water Table
Cone of Depression for
Lesser Pumping Rate
Cone of Depression for
Greater Pumping Rate
Drawdown
Radius of Influence



r_w = radius of well

b = height of aquifer

h_w = depth of water in the well

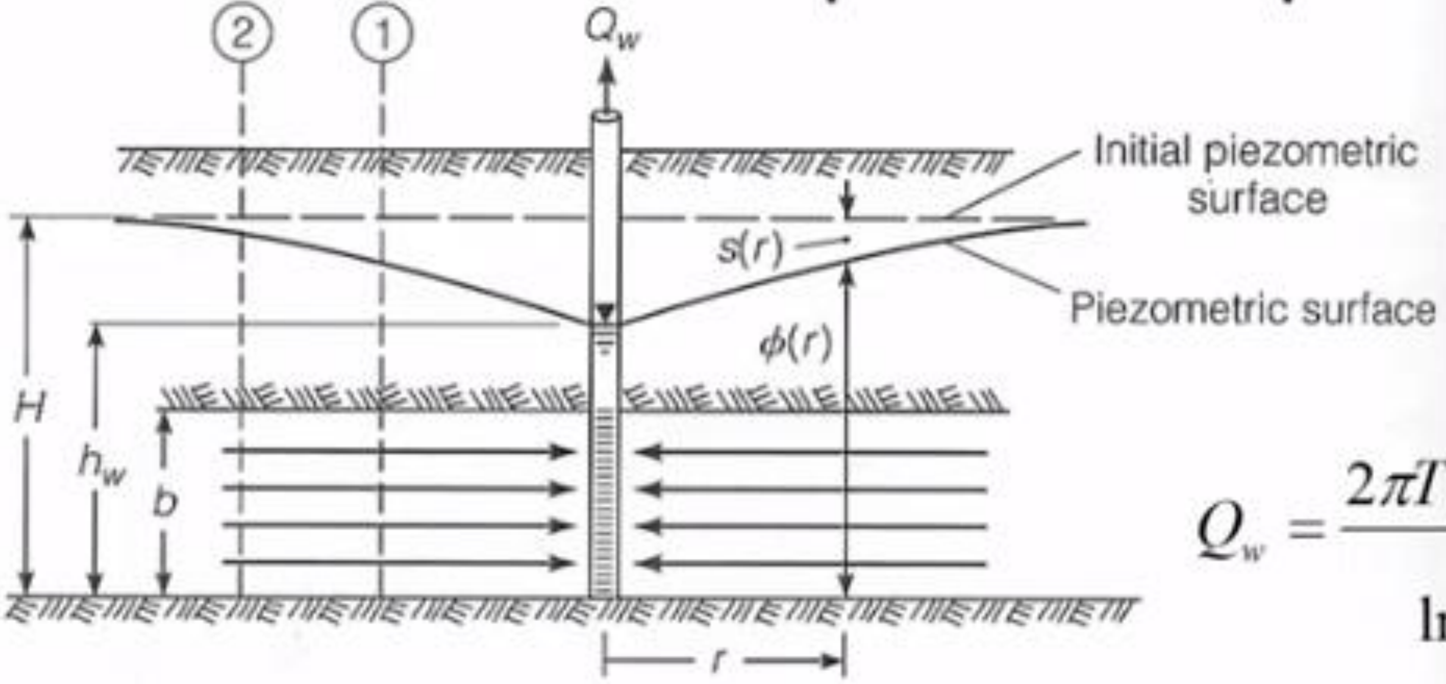
H = Initial piezometric surface

s(r) = drawdown at some radius, r

φ (r) = piezometric head at some radius, r

$$\phi(r) = h_w + \frac{Q_w}{2\pi K b} \ln\left(\frac{r}{r_w}\right)$$

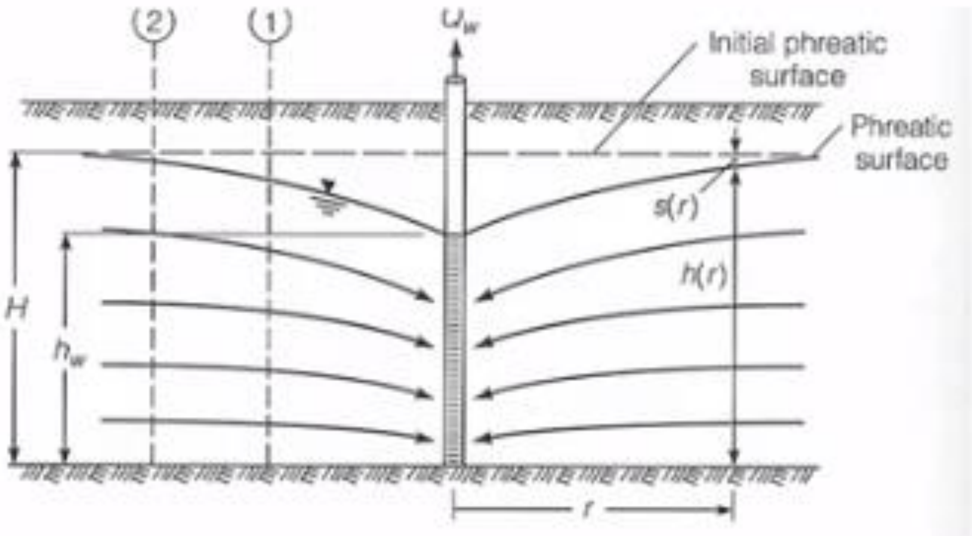
$$Q_w = \frac{2\pi T(h_2 - h_1)}{\ln\left(\frac{r_2}{r_1}\right)}$$



$$Q_w = \frac{2\pi T(h_2 - h_1)}{\ln\left(\frac{r_2}{r_1}\right)}$$

- A confined aquifer has hydraulic conductivity of 20 m/d, thickness of 6.6 m, and initial piezometric surface of 14.53 m above the lower confining layer.
  - What flowrate will cause the piezometric surface to be 13.85 m at a radius of 40 m and 14.31 at a radius of 85 m?
  - What is the water depth at the well if the well diameter is 0.50 m?

Steady Well Flow: Unconfined Aquifer



r_w = radius of well

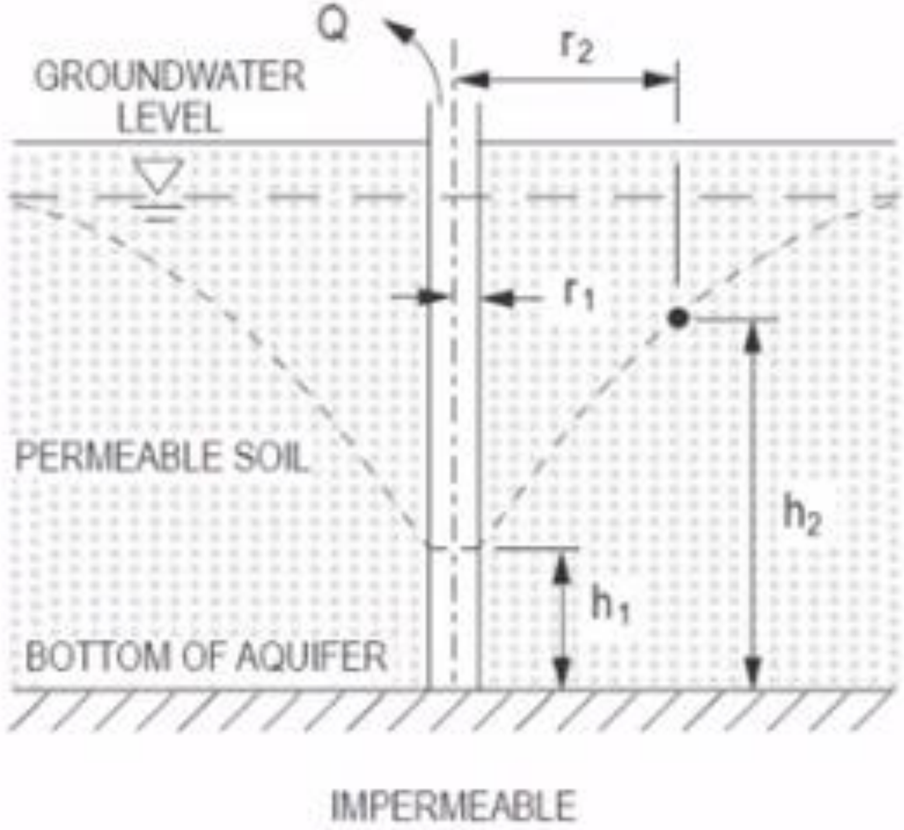
H = height of aquifer

h_w = depth of water in the well

H = Initial phreatic surface

s(r) = drawdown at some radius, r

$$Q_w = \frac{\pi K (h_2^2 - h_1^2)}{\ln\left(\frac{r_2}{r_1}\right)}$$

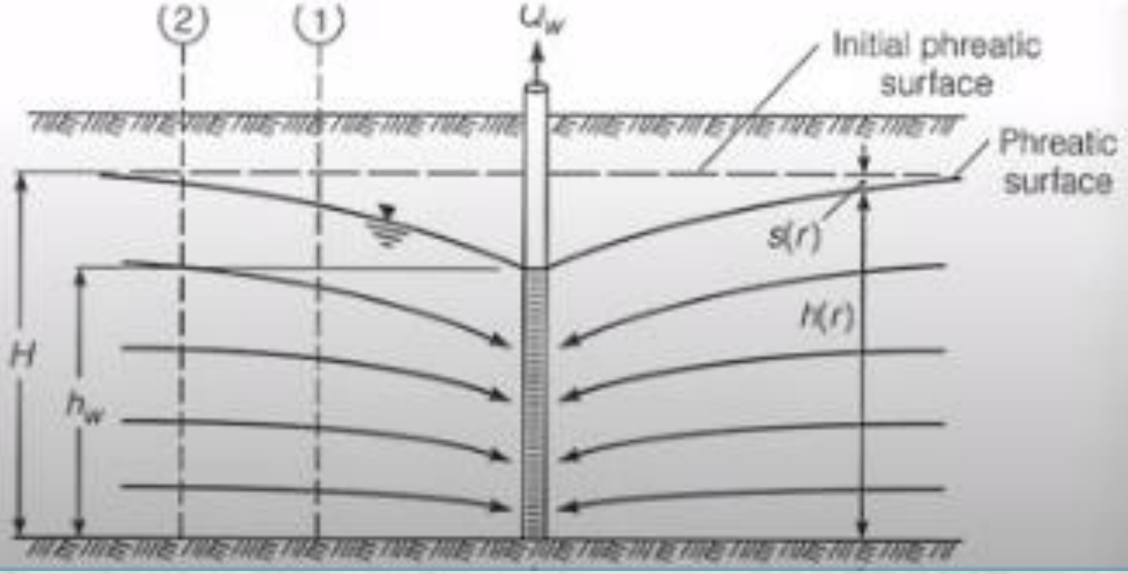


- Q = flow rate of water drawn from well (cfs)
- K = coefficient of permeability of soil = hydraulic conductivity (ft/sec)
- h_1 = height of water surface above bottom of aquifer at perimeter of well (ft)
- h_2 = height of water surface above bottom of aquifer at distance r_2 from well centerline (ft)
- r_1 = radius to water surface at perimeter of well, i.e., radius of well (ft)
- r_2 = radius to water surface whose height is h_2 above bottom of aquifer (ft)
- ln = natural logarithm
- Q/D_w = specific capacity
- D_w = well drawdown (ft)

Dupuit's Formula

$$Q = \frac{\pi K\left(h_2^2 - h_1^2\right)}{\ln\left(\frac{r_2}{r_1}\right)}, \text{where}$$

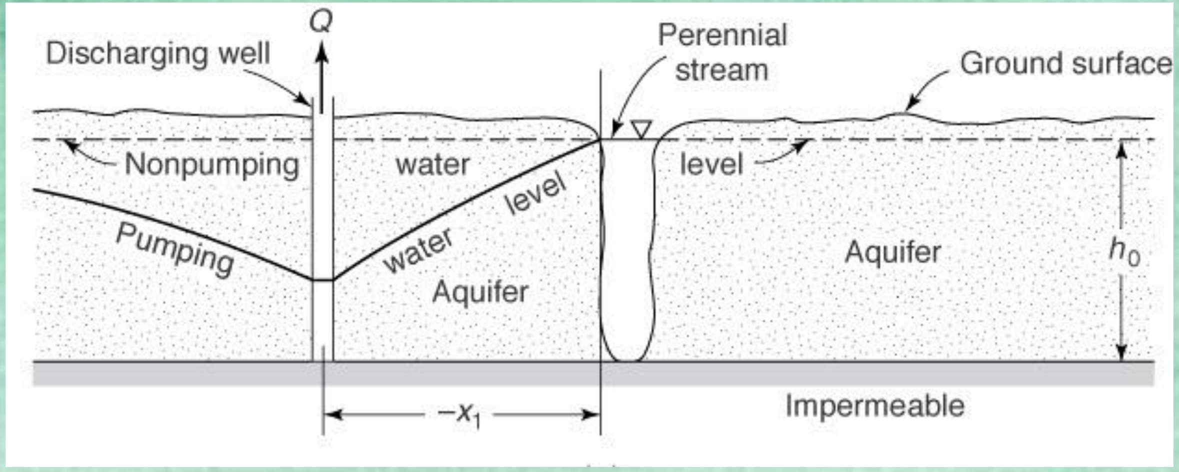
- A well pumps 0.4 m^3/s from an unconfined aquifer whose saturated thickness is 24 m. If the drawdown 50 m from the well is 1m and the drawdown 100 m from the well is 0.5 m, then:
  - (a) **calculate the hydraulic conductivity** of the aquifer.
  - (b) Determine the expected **drawdown** 5 m from the well.

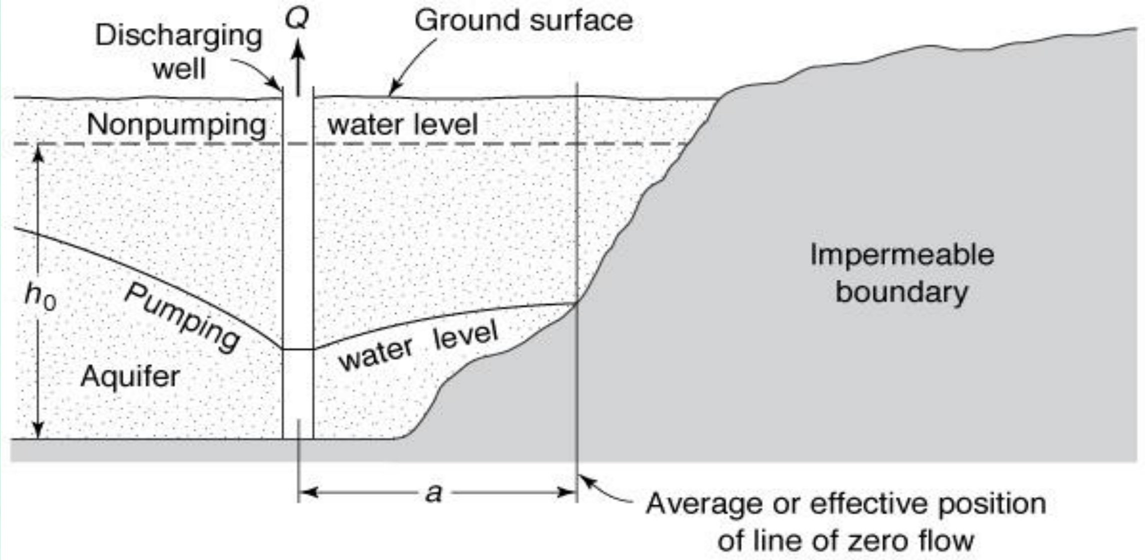


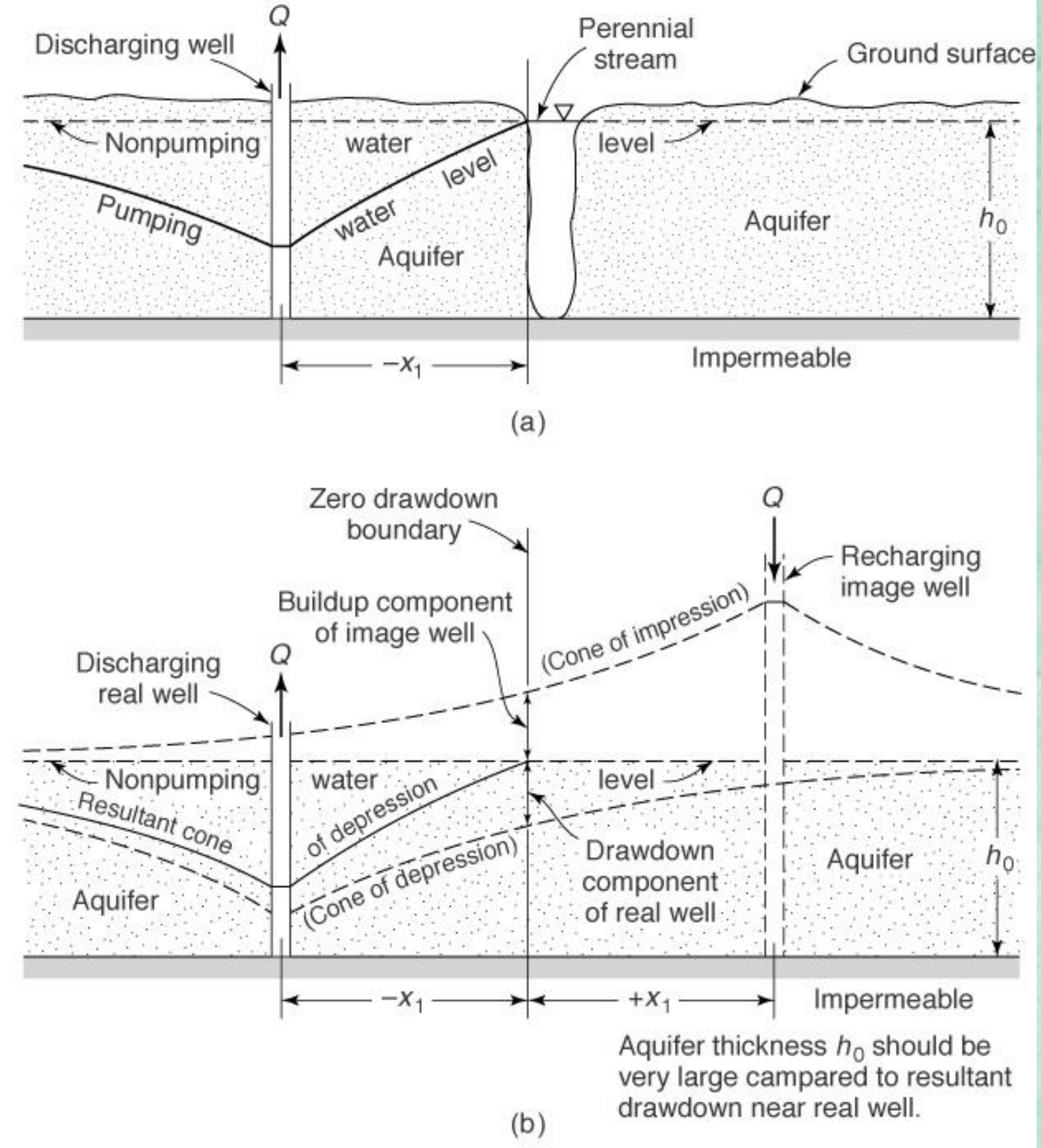
$$Q_w = \frac{\pi K (h_2^2 - h_1^2)}{\ln\left(\frac{r_2}{r_1}\right)}$$

دانشگاه علم و صنعت ایران

پمپاژ در نزدیکی مرزها







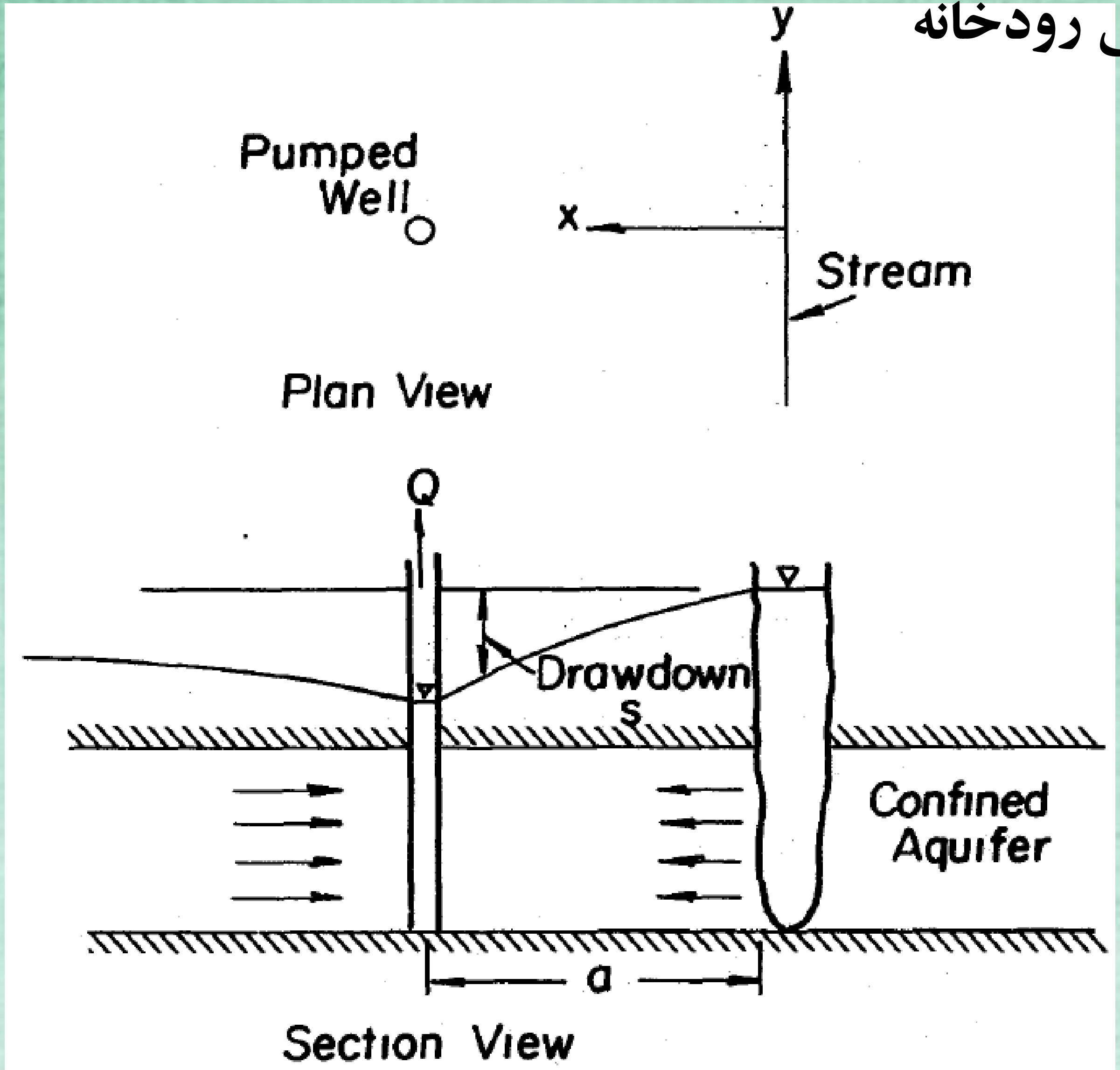
پمپاژ در نزدیکی رودخانه

فرضیات (؟)

شکل ۴-۱۱-جایگزینی چاه تغذیه با رودخانه در مسئله پمپاژ در نزدیکی رودخانه

اصل جمع آثار

پمپاژ در نزدیکی رودخانه

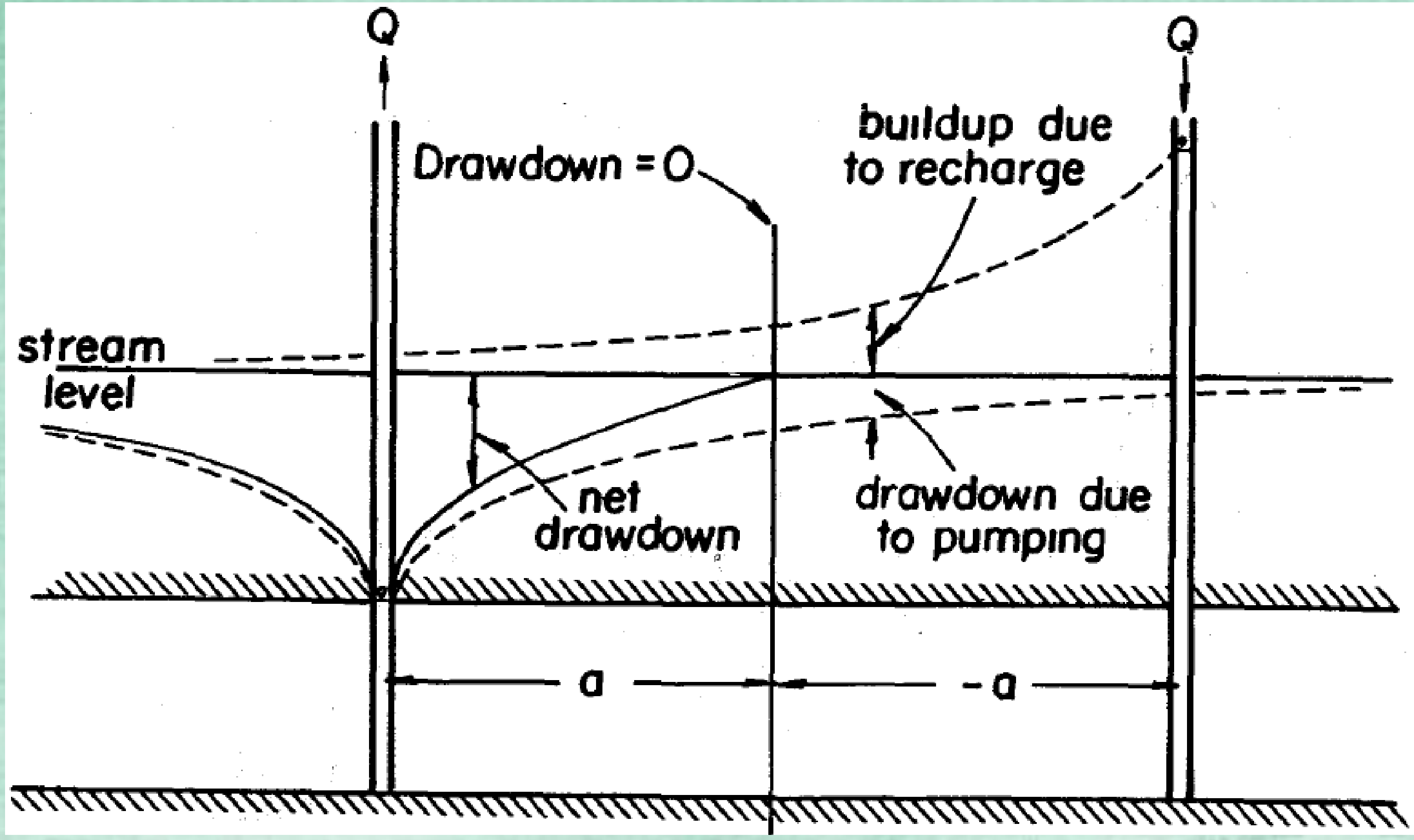


شکل ۴-۱۲- فرم ساده شده مسئله پمپاژ در نزدیکی رودخانه در آبخوان تحت فشار

اصل جمع آثار

$$s = \frac{Q}{2\pi T} \ln r_i/r = \frac{Q}{2\pi T} \ln\{ \frac{(a+x)^2+y^2}{(a-x)^2+y^2} \}^{\frac{1}{2}}$$

$$s_w = \frac{Q}{2\pi T} \ln(\frac{2a-r_w}{r_w}) \simeq \frac{Q}{2\pi T} \ln \frac{2a}{r_w}$$



شکل ۴-۱۳ افت و خیز حاصل از چاه های پمپاژ و تغذیه و افت خالص حاصل

EXAMPLE 4-4

Derive an equation for the family of equipotential lines for a pumped well located a perpendicular distance a from an infinitely long, fully penetrating stream with constant and horizontal water surface.

Solution:

From Eq. 4-35, the velocity potential function is

$$\Phi = \frac{-Q}{2\pi b} \ln\left\{\frac{(a+x)^2+y^2}{(a-x)^2+y^2}\right\}^{\frac{1}{2}} + \text{constant} \quad .$$

Putting the arbitrary constant equal to zero, noting that $\Phi=\Phi_i$ on an equipotential contour, and exponentiating yields

$$\left\{\frac{x^2+y^2+2ax+a^2}{x^2+y^2-2ax+a^2}\right\}^{-1} = \exp\left(\frac{4\pi b\Phi_i}{Q}\right) = c_i \quad ,$$

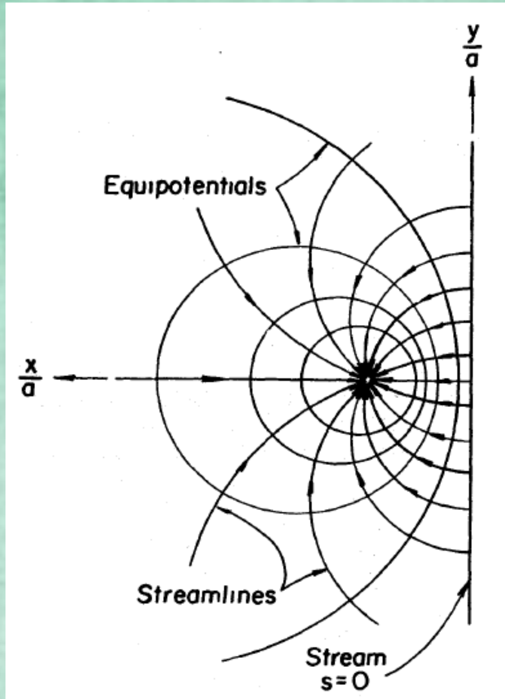
where c_i is a constant corresponding to Φ_i. Rearranging and completing the square results in

$$\left(\frac{x}{a} - \frac{1+c_i}{1-c_i}\right)^2 + \left(\frac{y}{a}\right)^2 = \left(\frac{1+c_i}{1-c_i}\right)^2 - 1 \quad ,$$

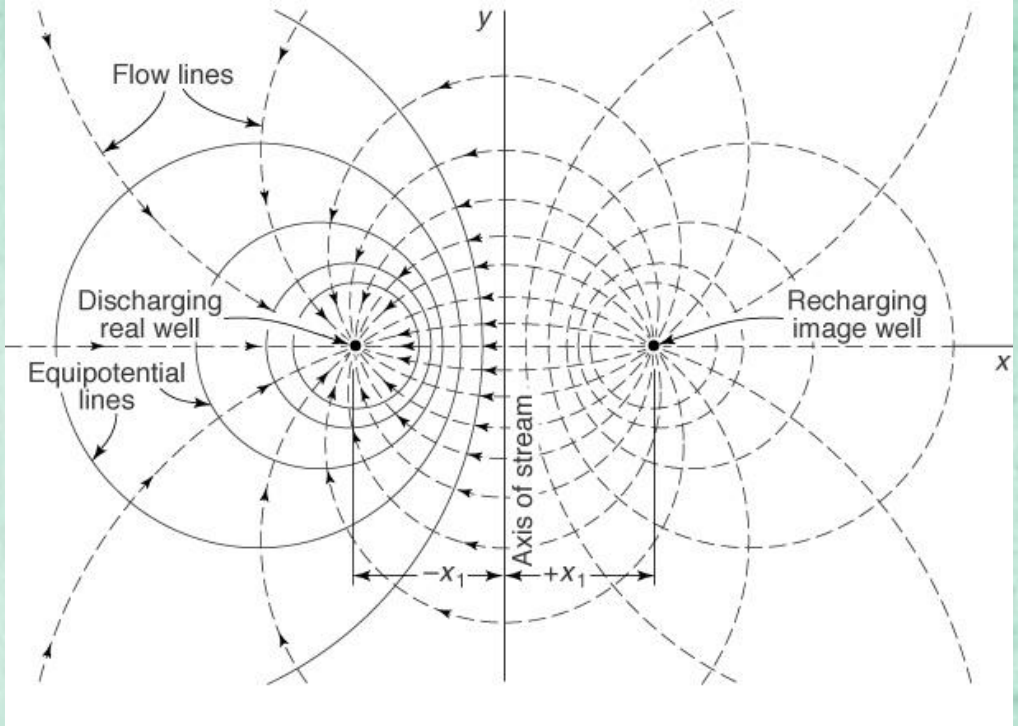
which represents a family of circles in the dimensionless coordinates $(x/a,\ y/a)$ with the centers at $x/a = (1+c_i)/(1-c_i)$, $y=0$ and radii $\{(1+c_i)^2/(1-c_i)^2 - 1\}^{\frac{1}{2}}$.

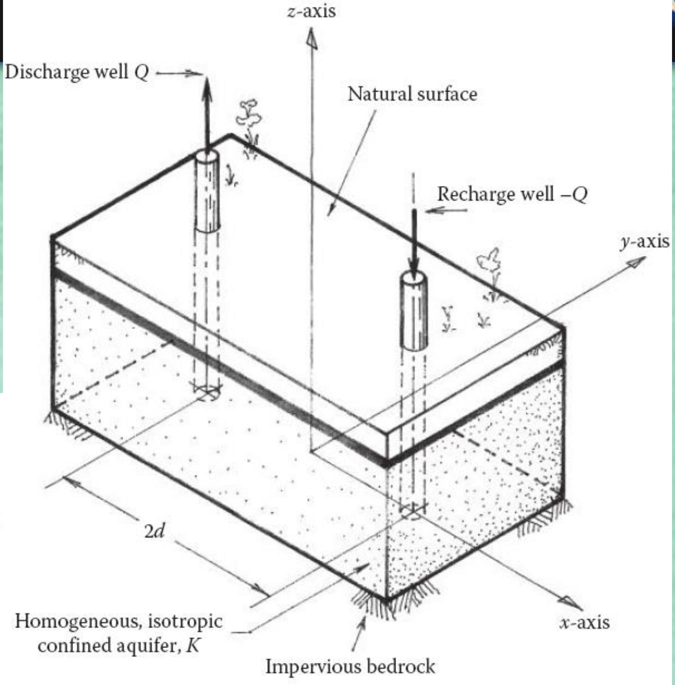
It is left for the student to show that the streamlines are also circles. The network of equipotential and streamlines is shown in Fig. 4-11.

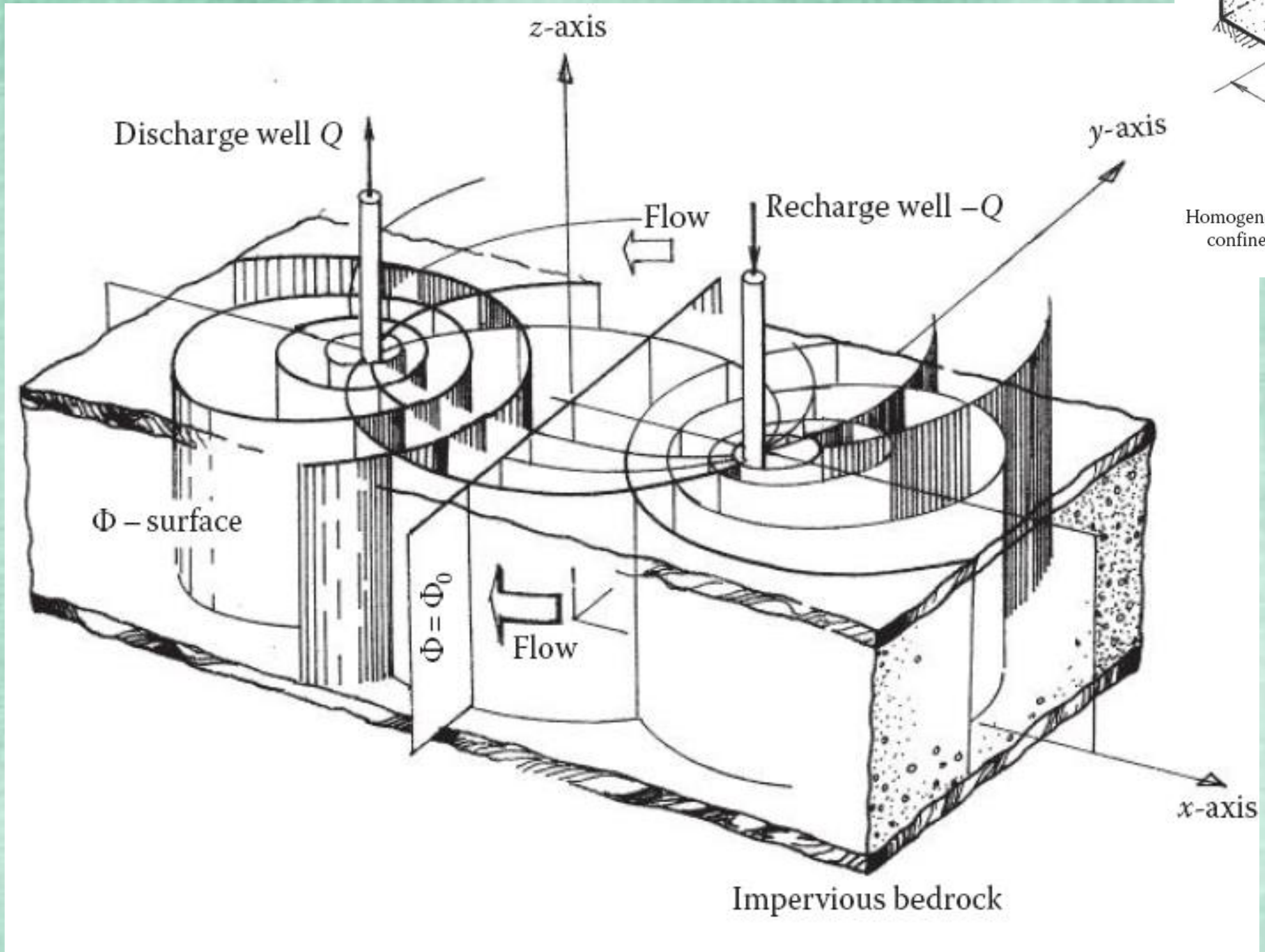
اصل جمع آثار



شکل ۴-۱۴- خطوط جریان
و هم پتانسیل در مثال







شکل ۴-۱۴- سطوح جریان
و هم پتانسیل در مثال

EXAMPLE 4-5

Verify that the total discharge from the stream to the aquifer is equal to the well discharge for the steady flow depicted in Figs. 4-7 and 4-8.

Solution:

The discharge dQ_s from the stream into the aquifer on a reach of length dy follows from Darcy's law:

$$dQ_s = q_x b dy = T \left.\frac{\partial s}{\partial x}\right|_{x=0} dy \quad .$$

The drawdown is

$$s = \frac{Q}{2\pi T} \ln \frac{r_i}{r} = \frac{Q}{4\pi T} \ln\left\{ \frac{(x+a)^2+y^2}{(a-x)^2+y^2} \right\} \quad ,$$

which follows from Eq. 4-35 and the coordinate system indicated in Fig. 4-7. Calculation of the partial derivative of s with respect to x yields

$$\frac{\partial s}{\partial x} = \frac{Q}{4\pi T} \frac{(a-x)^2+y^2}{(x+a)^2+y^2} \left[\frac{2\{(a-x)^2+y^2\}(x+a) + 2\{(x+a)^2+y^2\}(a-x)}{\{(a-x)^2+y^2\}^2} \right] \quad .$$

At x=0, the result is

$$\left.\frac{\partial s}{\partial x}\right|_{x=0} = \frac{Q}{\pi T} \left(\frac{a}{a^2+y^2} \right) \quad ,$$

and the differential discharge from reach dy becomes

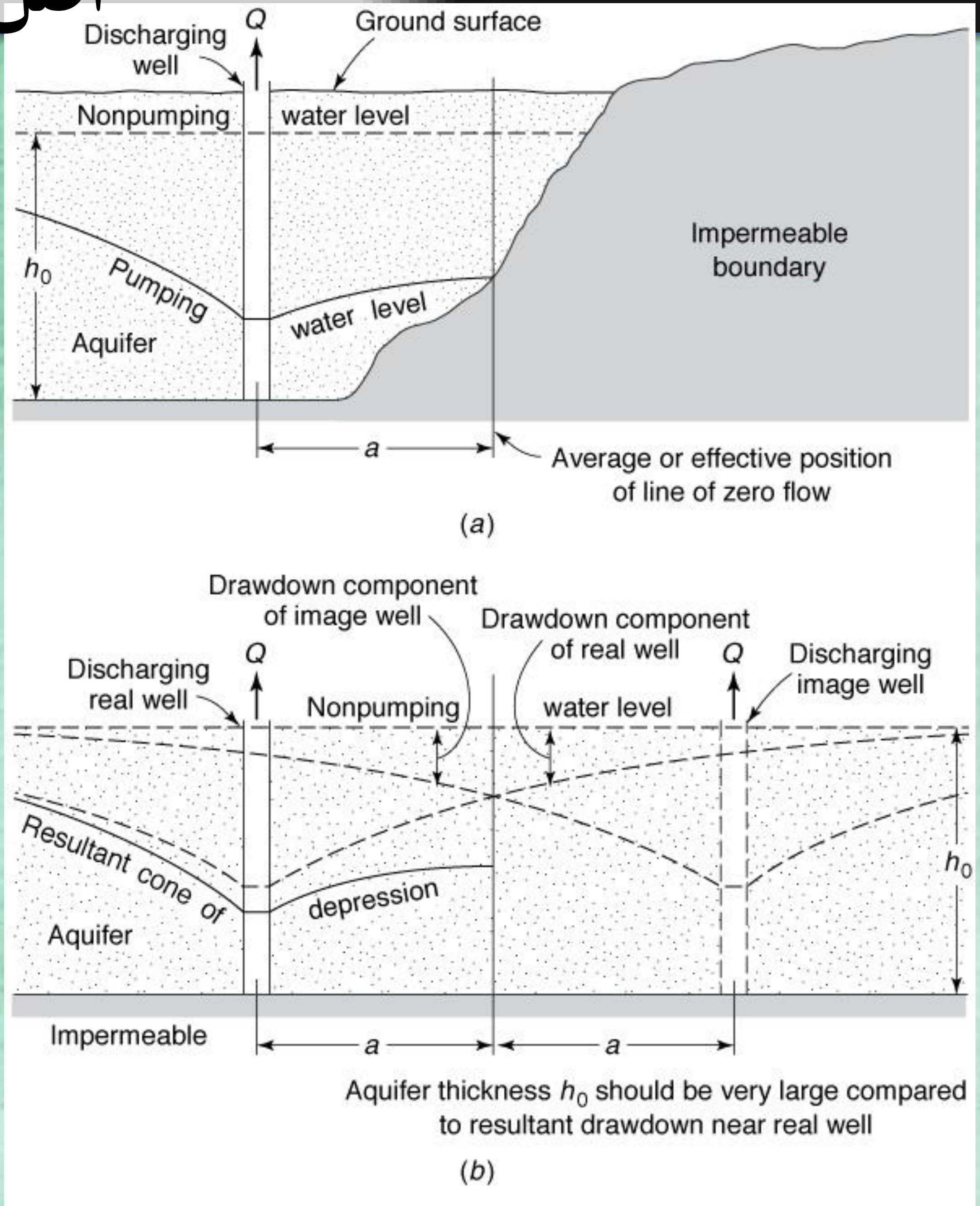
$$dQ_s = \frac{Q}{\pi} \frac{a}{a^2+y^2} dy \quad .$$

The discharge from the stream to the aquifer is obtained by integrating over the reach of stream extending from $-\infty$ to ∞:

$$Q_s = \frac{Qa}{\pi} \int_{-\infty}^{\infty} \frac{dy}{a^2+y^2} = \frac{2Q}{\pi} \tan^{-1} \frac{y}{a} \Big]_0^{\infty} \quad ,$$

from which $Q_s = Q$ as required.

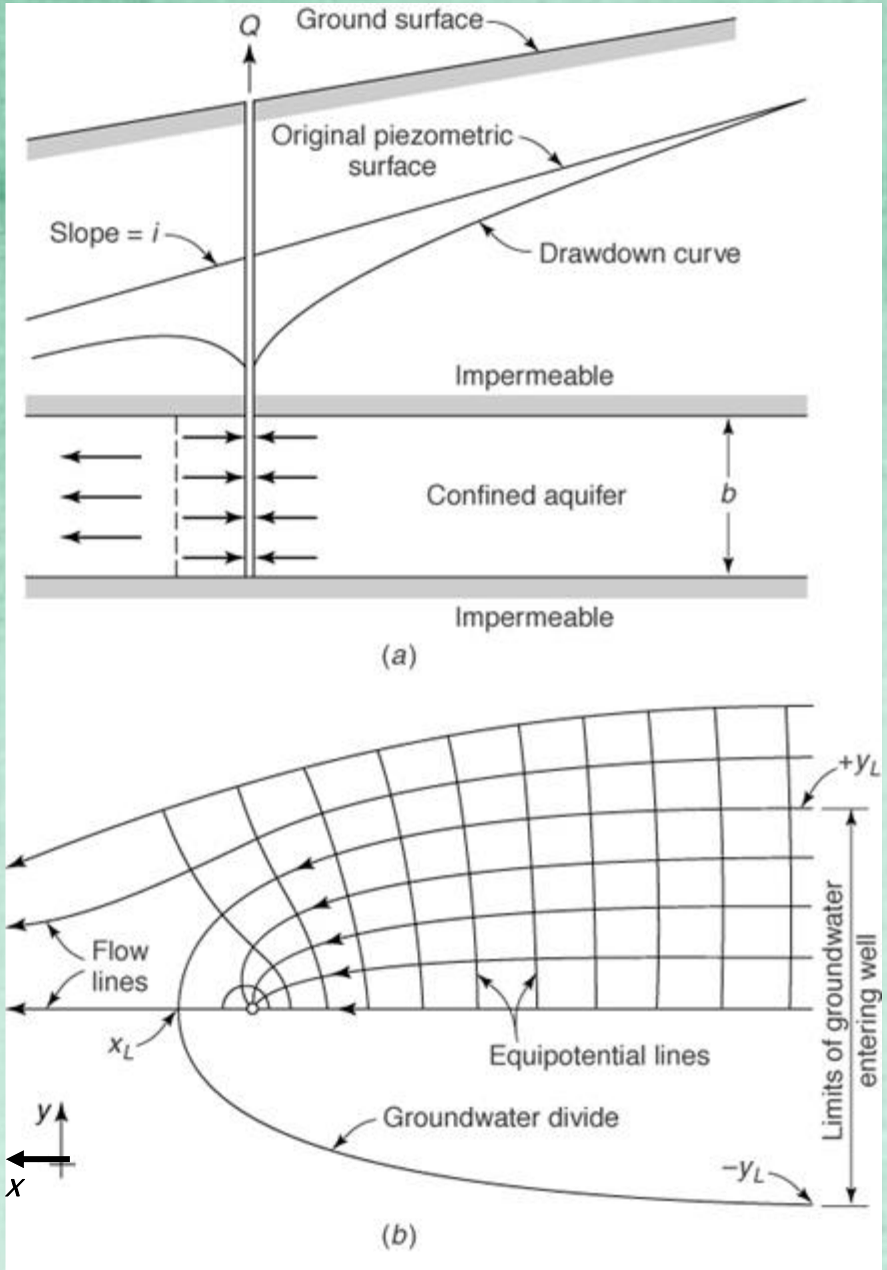
اصل جمع آثار



پمپاژ در نزدیکی مرز نفوذناپذیر

شکل ۴-۱۵-افت حاصل از چاه های پمپاژ
و تصویر و افت خالص حاصل

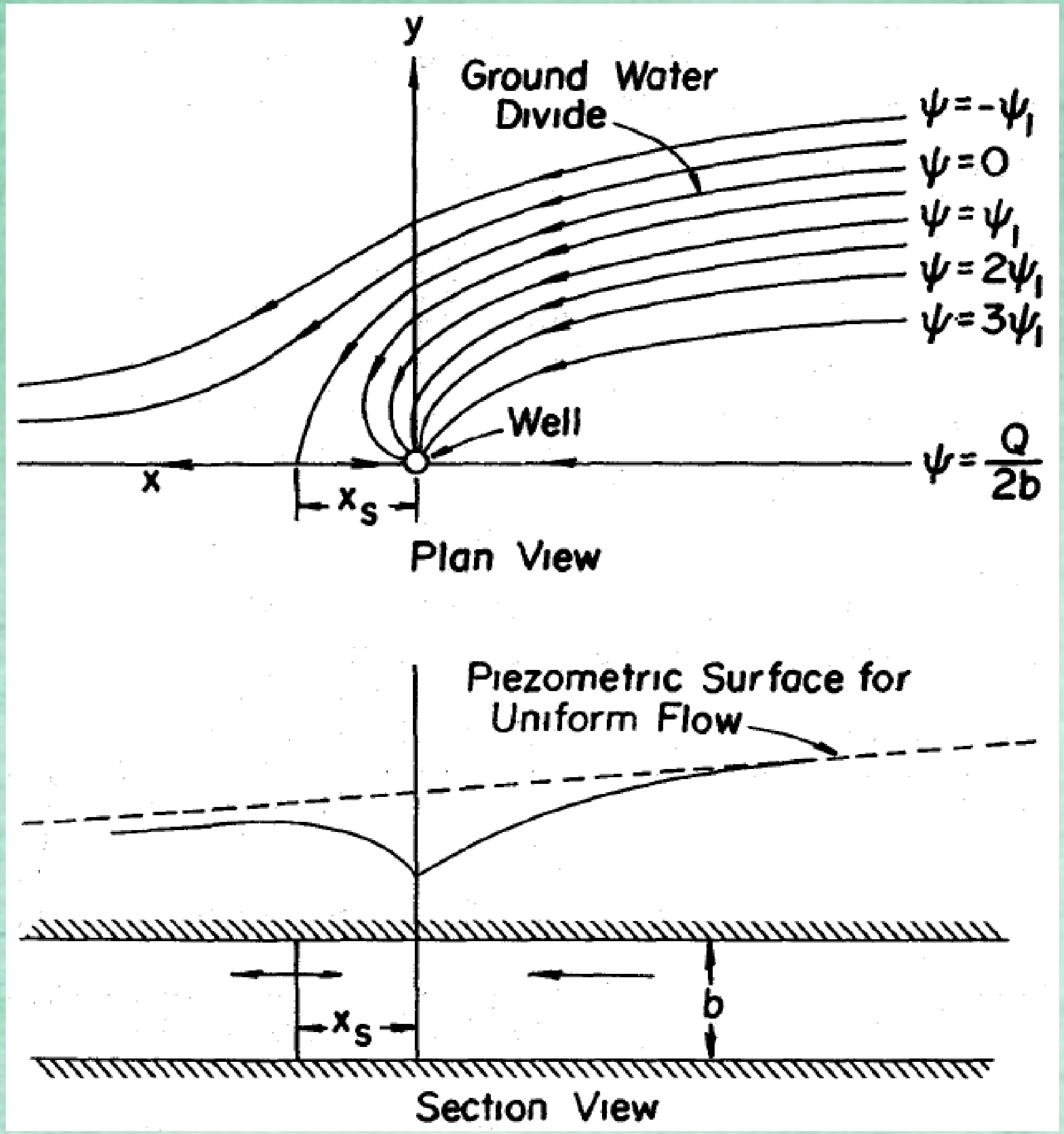
چاه در جریان یکنواخت



شکل ۴-۱۶- الگوی خطوط جریان و شبکه جریان در جریان ترکیبی چاه و جریان یکنواخت

اصل جمع آثار

چاه در جریان یکنواخت



شکل ۴-۱۶- الگوی خطوط جریان و شبکه جریان در جریان ترکیبی چاه و جریان یکنواخت

چاه در جریان یکنواخت

A Pumped Well In Uniform Flow

Superposition can be used to determine the flow pattern for a well placed in an aquifer in which one-dimensional uniform flow occurs. The equations for the streamlines are obtained by adding the ψ functions for uniform and radial flow. The stream function for one-dimensional, uniform flow in the x-direction is

$$\psi = -q_x y + \text{constant} = Kiy + \text{constant} \quad , (4\text{-}37)$$

where i=dh/dx has been introduced for simplicity in notation. Note that i may be positive or negative depending upon the coordinate system selected. Adding ψ from Eq. 4-37 and 4-26 yields

$$\psi = Kiy + \frac{Q}{2\pi b}\delta + \text{constant} \qquad (4\text{-}38)$$

which is the stream function for a well in uniform flow. The flow pattern in the upper-half plane is shown in Fig. 4-12. The flow is symmetrical about the x-axis.

Recall that the numerical difference between any two values of constant ψ is equal to the discharge per unit of aquifer thickness between the two streamlines. Selecting the arbitrary constant in Eq. 4-38 equal to zero, it is evident that ψ is Q/2b when y=0 and $\delta=\pi$. Thus, the entire negative x-axis is a streamline for which ψ=Q/2b. It follows that one-half of the well discharge must flow between the streamlines ψ=Q/2b and ψ=0.

Furthermore, the streamline for which $\psi=0$ must separate flow that eventually contributes to well discharge from flow that bypasses the well. The streamline $\psi=0$ is called the ground-water divide (Fig. 4-12). The relationship between the x and y coordinates of all points on the ground-water divide is

چاه در جریان یکنواخت

$$Kiy = -\frac{Q}{2\pi b}\delta \qquad (4\text{-}39)$$

where $\delta=\tan^{-1}(y/x)$ for $x\geq 0$ and $\delta=\pi-\tan^{-1}|y/x|$ for $x\leq 0$. For the coordinate system shown in Fig. 4-12, the slope i of the piezometric surface in uniform flow is negative.

The point at which the ground-water divide streamline crosses the x-axis downstream of the well is called a stagnation point because the Darcy velocity is zero there. Water on the x-axis upstream of the stagnation point moves toward the well, while water on the x-axis downstream of the stagnation point moves away from the well. The coordinate x_s of the stagnation point is obtained from

$$x_s = \lim_{y\to 0}\left[\frac{y}{\tan\left(\frac{-2\pi Tiy}{Q}\right)}\right] = -\frac{Q}{2\pi Ti} \quad . \qquad (4\text{-}40)$$

Far upstream of the well, (as $x\to-\infty$ and $\delta\to\pm\pi$) Eq. 4-39 yields

$$y = \pm\frac{Q}{2Ti} \qquad (4\text{-}41)$$

which is the half-width of that portion of the aquifer in which flow contributes to the well discharge.

The distribution of piezometric head is calculated by adding the heads for radial and uniform flow. From Eqs. 4-12 and 4-23

$$\Phi = -q_x x + \frac{Q}{2\pi b}\ln r + \text{constant} \qquad (4\text{-}42)$$

or

$$h = ix + \frac{Q}{4\pi T}\ln(x^2+y^2) + \text{constant} \quad . \qquad (4\text{-}43)$$

چاه در جریان یکنواخت

(برای مطالب تکمیلی به فایل تئوری پتانسیل مراجعه شود.)

EXAMPLE 4-7

Contaminated water from several waste-holding ponds seeps into an aquifer in which uniform flow exists. A well is to be constructed 300 m down gradient from the ponds in an attempt to intercept the contaminated ground water. The slope of the piezometric surface is -0.022 and the transmissivity is 0.013 m^2/s. The width of contaminant source area is 200 m, measured perpendicular to the direction of ground water flow (Fig. 4-13). Compute the well discharge required to intercept the contaminated ground water, assuming no dispersion. Assume further, that the discharge of contaminated water into the aquifer is small compared to the discharge through the aquifer.

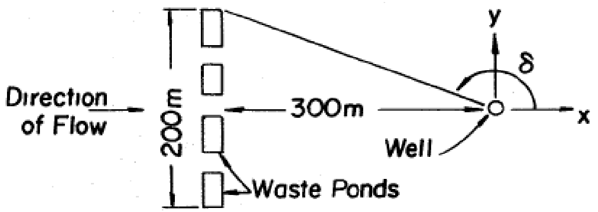


Figure 4-13. An interceptor well downstream of a source of ground-water contamination.

Solution:

The significance of the second assumption in the problem is that the discharge of contaminated water is too small to significantly influence the pattern of flow and that the quantity of contaminated water added does not appreciably alter the discharge per unit area in the aquifer. Provided lateral dispersion is negligible, the contaminated water will be intercepted if the $\psi=0$ streamline passes through the extremeties of the line of waste ponds. The coordinates of one end of the line on which contamination occurs are (-300,100). From Eq. 4-39,

$$Q = -\frac{2\pi T i y}{\delta} ,$$

wherein δ is the angle indicated in Fig. 4-13. Hence,

$$\delta = \pi - \tan^{-1}\left(\frac{100}{300}\right)$$

$$= 2.82 \text{ radians.}$$

The required discharge is

$$Q = \frac{-2\pi(0.013)(-0.022)(100)}{2.82} = 0.064 \text{ m}^3/\text{s} \quad .$$

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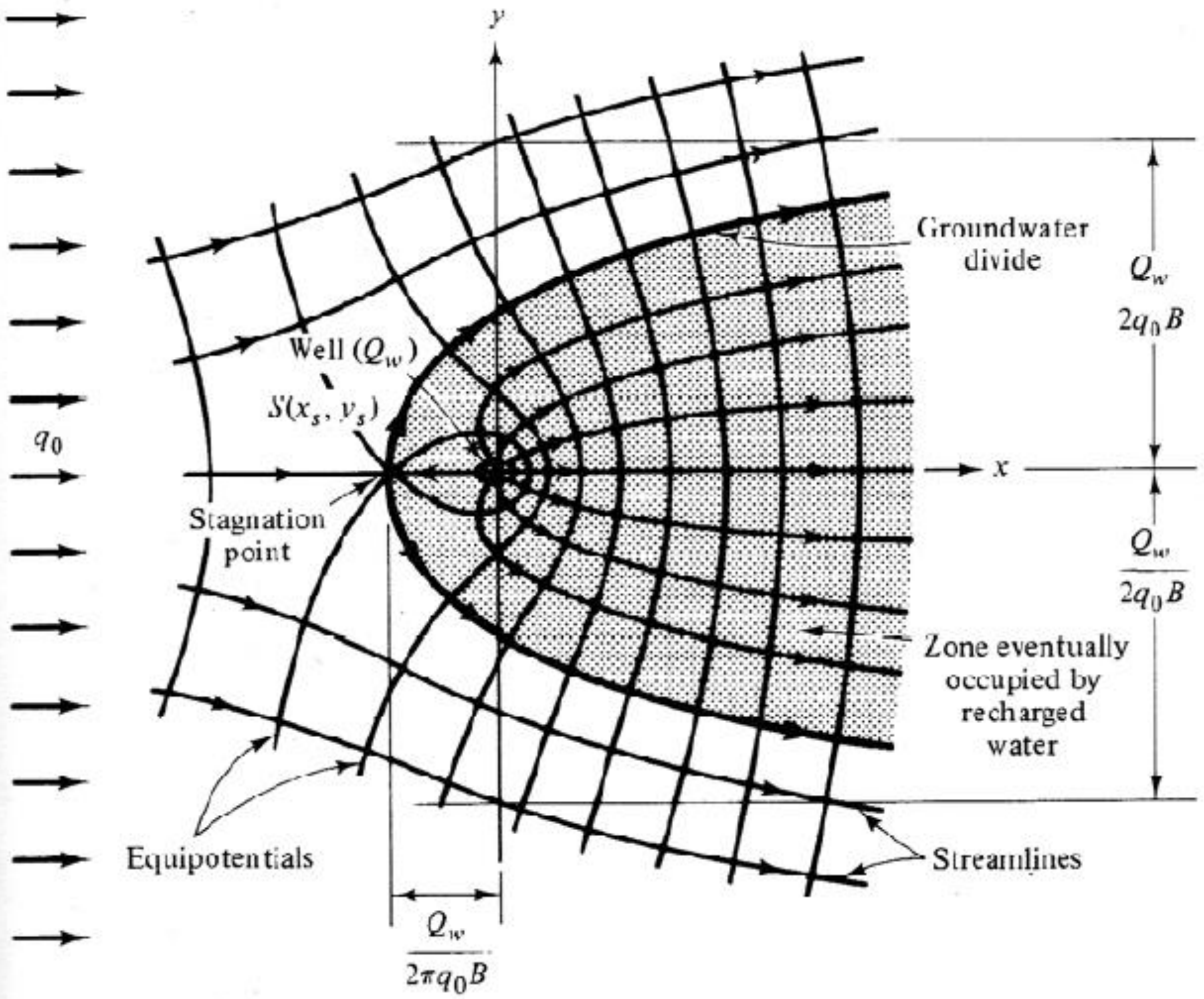

Figure 8 30 A single recharging well in uniform flow.

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4.4 FLOW NETS

Thus far, analytic methods for obtaining solutions to the Laplace equation have been presented. It is apparent from the previous discussions that boundary conditions must be relatively simple in order to obtain analytic solutions even using superposition. Another solution technique which may be used is a graphical method known as the flow net.

Flow Nets In Homogeneous Aquifers

A flow net consists of a network of equipotential lines and the corresponding orthogonal streamlines (Fig. 4-11). From a properly constructed flow net one may obtain distribution of heads, discharges, areas of high (or low) velocities, and the general flow pattern. In addition, flow nets offer an excellent means of gaining insight into the general characteristics of ground water flow.

Referring to Fig. 4-15, the flow channel between adjacent streamlines is called a *streamtube*. The discharge in the streamtube per unit width perpendicular to the plane of the figure is

$$\Delta Q = \psi_2 - \psi_1 = \Delta\psi \quad (4\text{-}45)$$

and the Darcy velocity is

$$q = \Delta Q/\Delta B = \Delta\psi/\Delta B \quad .(4\text{-}46)$$

Darcy's Law gives, for this figure,

$$q = -\frac{K\Delta h}{\Delta \ell} = -\left(\frac{\Phi_2 - \Phi_1}{\Delta \ell}\right) = \frac{\Delta\Phi}{\Delta \ell} \quad (4\text{-}47)$$

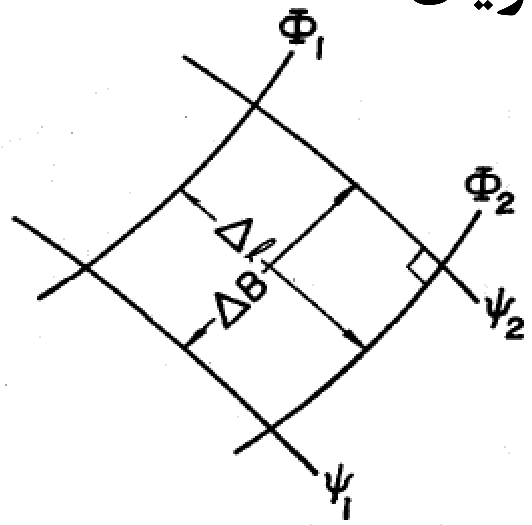

Figure 4-15. A flow net element for two-dimensional flow.

where $\Phi = Kh$. If the flow net is drawn so that $\Delta \ell$ is equal to ΔB, then the flow net reduces to a network of "squares" which can be easily identified by visual means. In addition, since continuity must be satisfied, then

$$q = \Delta\psi/\Delta B = \Delta\Phi/\Delta \ell \quad (4\text{-}48)$$

and the increment $\Delta\psi$ must be equal to $\Delta\Phi$.

The first step in the solution of a flow problem by flow net construction is to draw the two-dimensional flow domain to scale so that all boundary locations, wells, etc. are in the correct positions, relative to one another. A trial and error procedure is used to sketch the flow net. The following rules aid in minimizing the number of trials needed to construct a proper flow net:

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a) Equipotential contours and streamlines are orthogonal at points of intersection.

b) The flow net is constructed so that the spacing ΔB between streamlines is equal to the spacing $\Delta \ell$ between equipotential lines for each element of the net. In this way the flow net is constructed as a set of curvilinear squares of appropriate sizes.

c) All boundary conditions must be satisfied by the net, e.g., constant head boundaries or impermeable boundaries.

d) Take advantage of apparent symmetry (if any) by beginning the flow net construction around lines of symmetry.

e) Use no more than 4 or 5 streamtubes in the first trials.

The total discharge through the aquifer is (by Eqs. 4-45, 4-46, and 4-47)

$$Q = n_s \Delta Q = n_s \Delta\psi = n_s \Delta\Phi = n_s K \Delta h \qquad (4\text{-}49)$$

where n_s is the number of streamtubes and Δh is the head loss along the streamtube for one element. The head loss along a streamtube for each element is given by

$$\Delta h = \frac{H_t}{n_\ell} \qquad (4\text{-}50)$$

where H_t is the total head loss determined from the boundary conditions for a streamtube and n_ℓ is the number of equipotential drops determined from the flow net. From Eqs. 4-49 and 4-50, the discharge is

$$Q = K \frac{n_s}{n_\ell} H_t \quad . \tag{4-51}$$

For a particular problem the ratio n_s/n_ℓ is a constant. Only by coincidence is n_ℓ an integer when an integer number of streamtubes is used at the outset.

The above procedures and equations permit one to determine discharges and potential distribution in cases in which the aquifer is homogeneous. Flow beneath impermeable dams and under cut-off walls are common cases that are analyzed by these methods.

EXAMPLE 4-9

Using the flow net method, estimate the discharge under the sheet piling shown in Fig. 4-16 for $K=2.5\times10^{-5}$ m/s and $H_t=2$ m. Note that the sheet piling penetrates to 1/2 of the aquifer thickness.

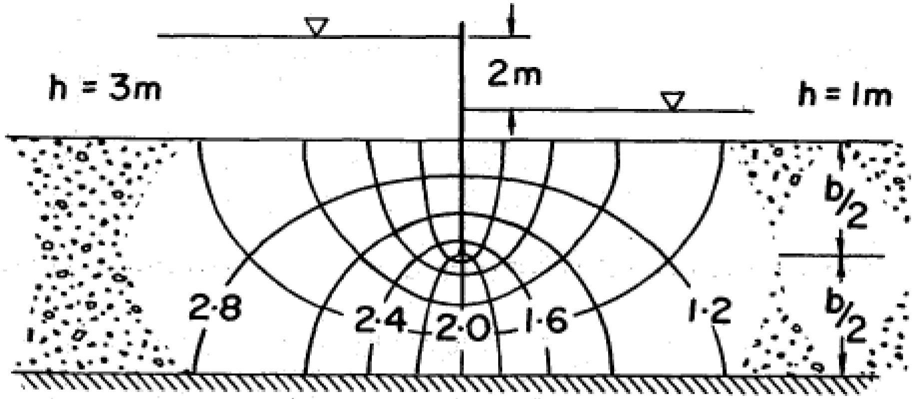

Figure 4-16. Flow net for seepage under a sheet pile (Adapted from Polubarinova-Kochina, 1962).

$$Q = K \frac{n_s}{n_\ell} H_t = 2.5\times10^{-5}\left(\frac{5}{10}\right)(2) = 2.5\times10^{-5}\ m^2/s$$

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EXAMPLE 4-10

Calculate the seepage discharge beneath the dam and cut-off wall shown in Fig. 4-17. The hydraulic conductivity is 15 m/d and H_t=10 m.

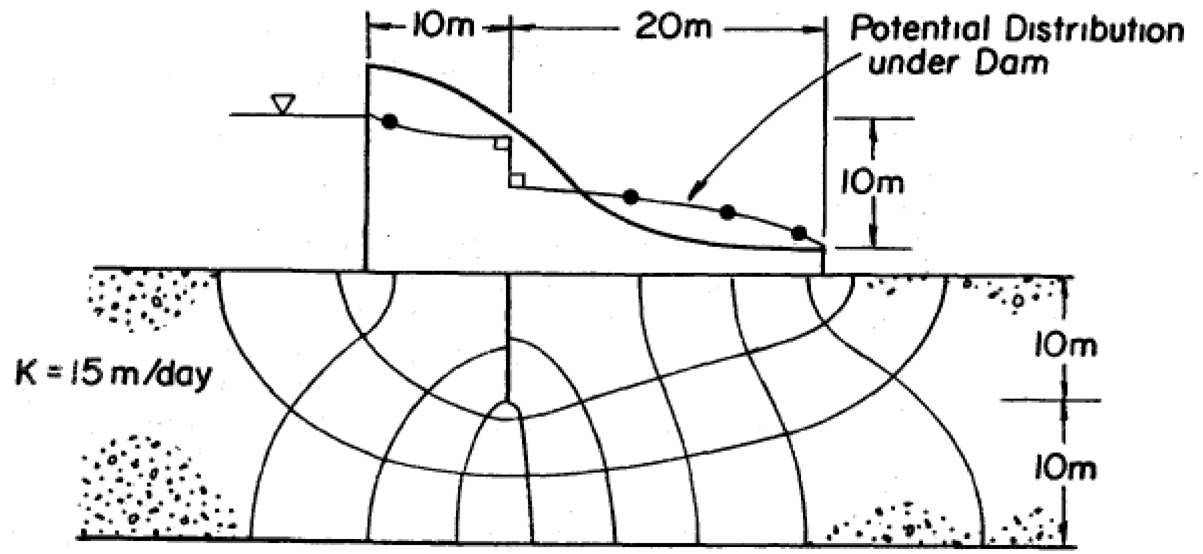

Figure 4-17. Flow net for seepage beneath a dam with a cut-off wall.

Solution:

A flow net is also shown with 9 equipotential drops and 3 stream channels. The discharge per unit width perpendicular to the plane of the cross-section is

$$Q = \left(\frac{n_s}{n_\ell}\right) KH_t = \left(\frac{3}{9}\right)(15)(10) = 50 \frac{m^3}{m \cdot day} .$$

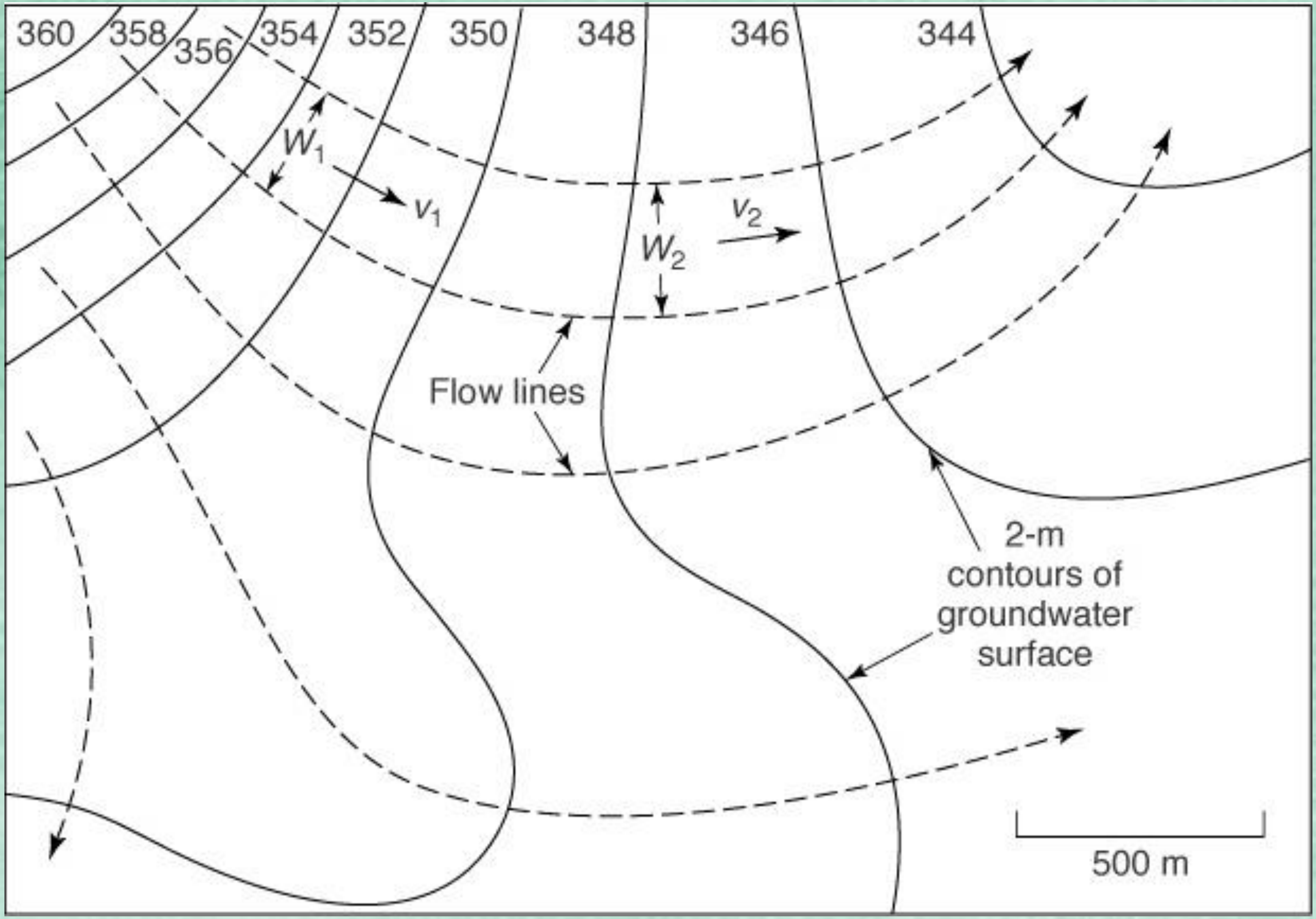

Figure 3.6.5. Contour map of a groundwater surface showing flow lines.

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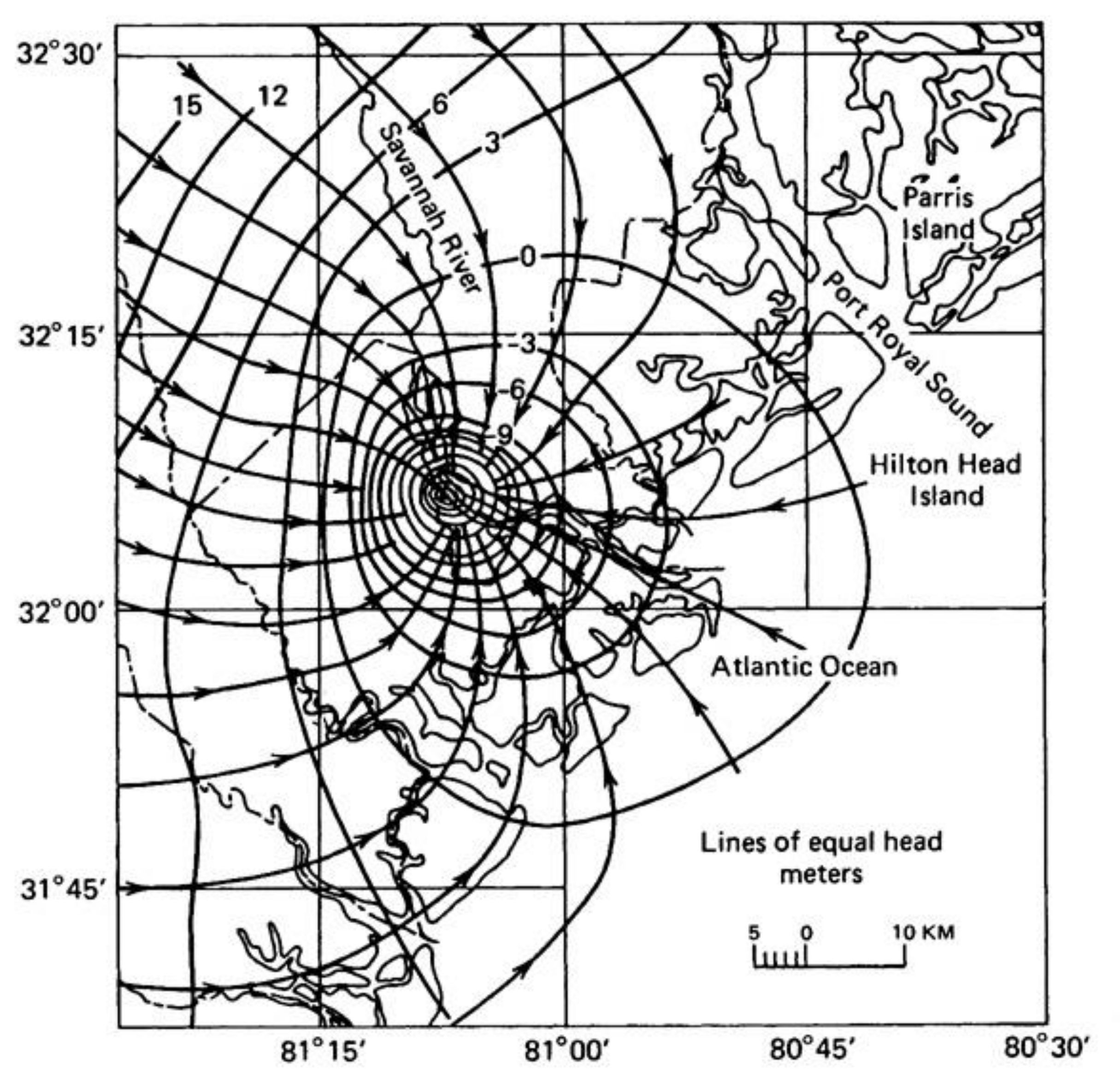

Figure 3.6.6. Contour map of the piezometric surface near Savannah, Georgia, 1957, showing closed contours resulting from heavy local groundwater pumping (after *U.S.G.S. Water-Supply Paper* 1611).

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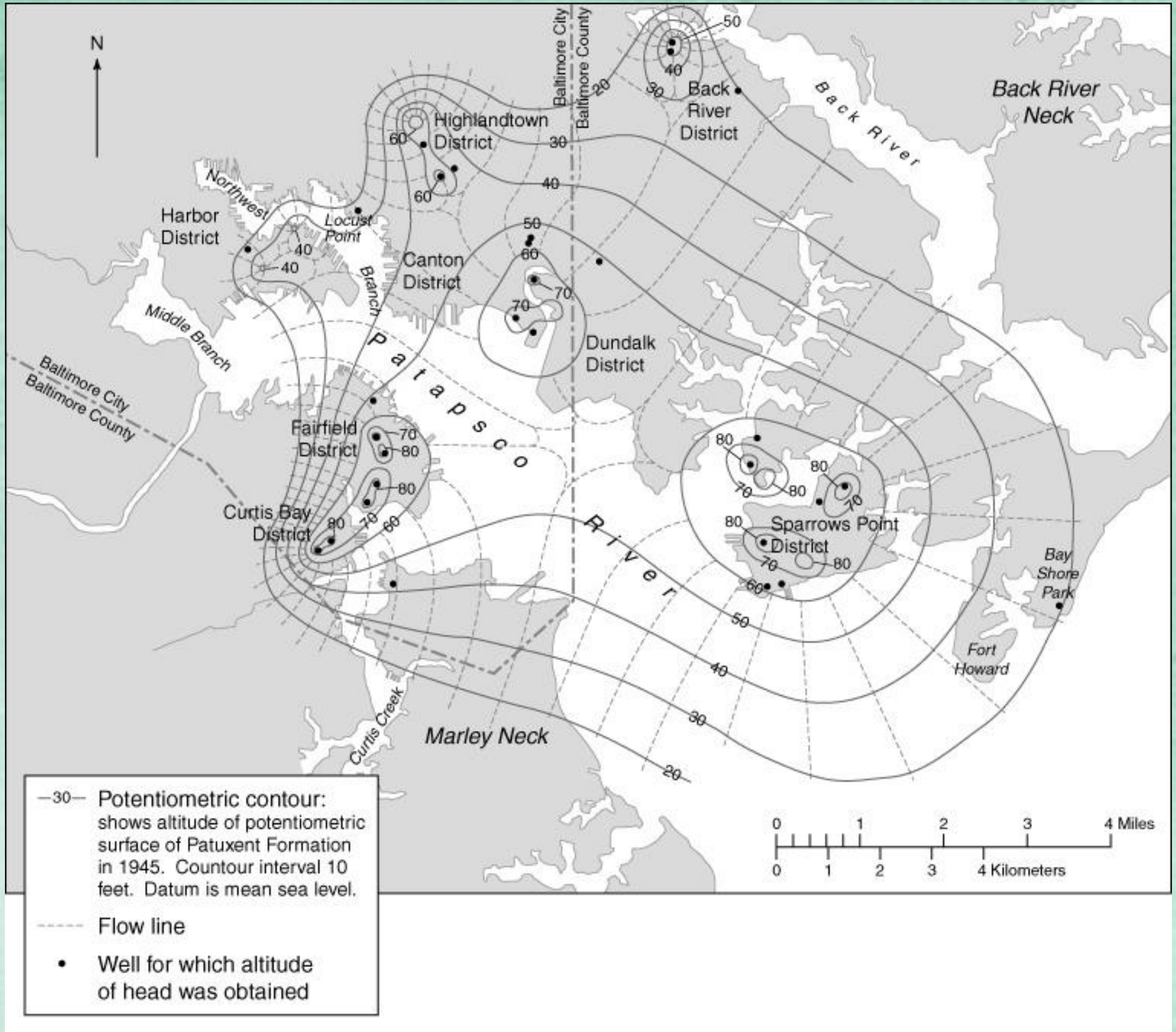
Figure 3.6.8. Map of Baltimore industrial area. Maryland, showing potentiometric surface in 1945 and generalized flow lines in the Patuxent Formation. From Bennett and Meyer[9] (as presented in Lohman[66]).



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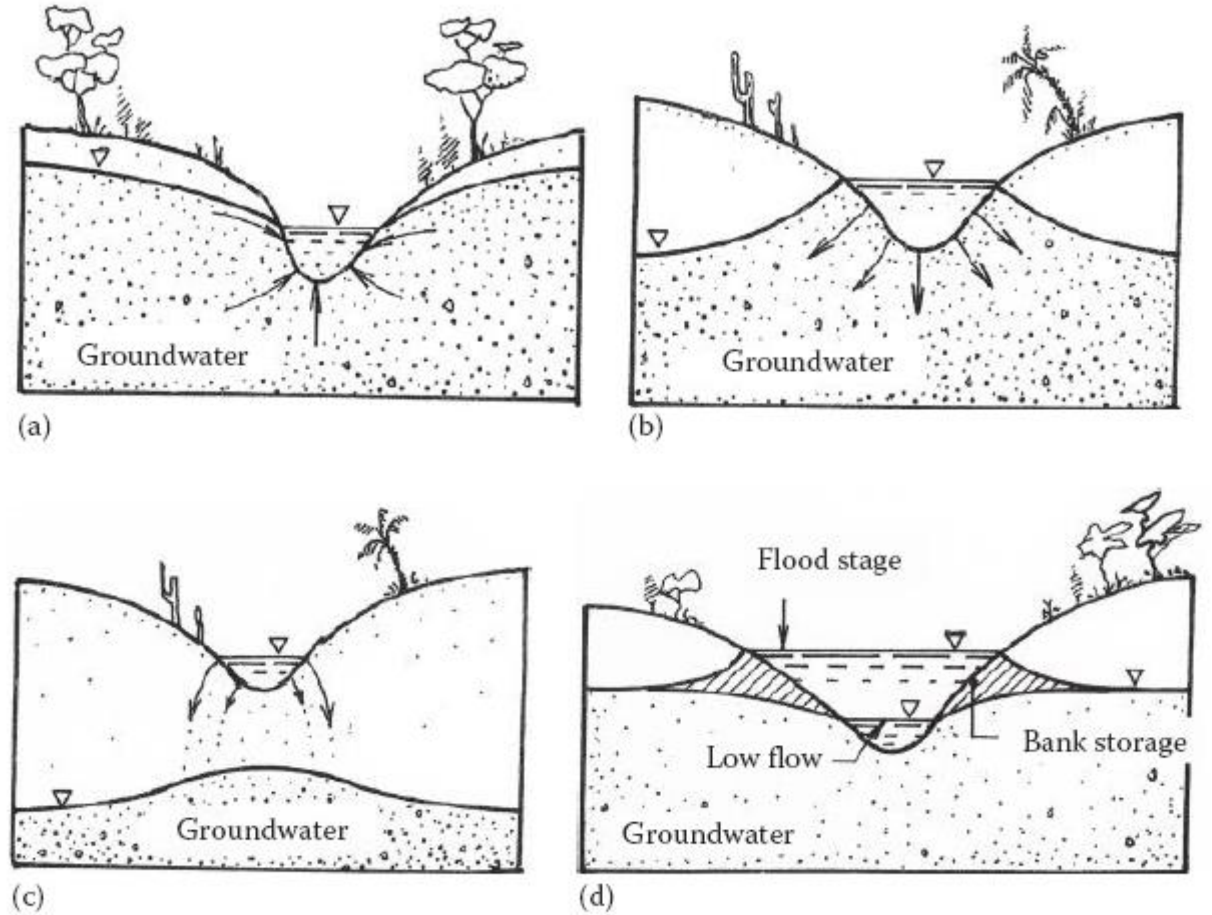
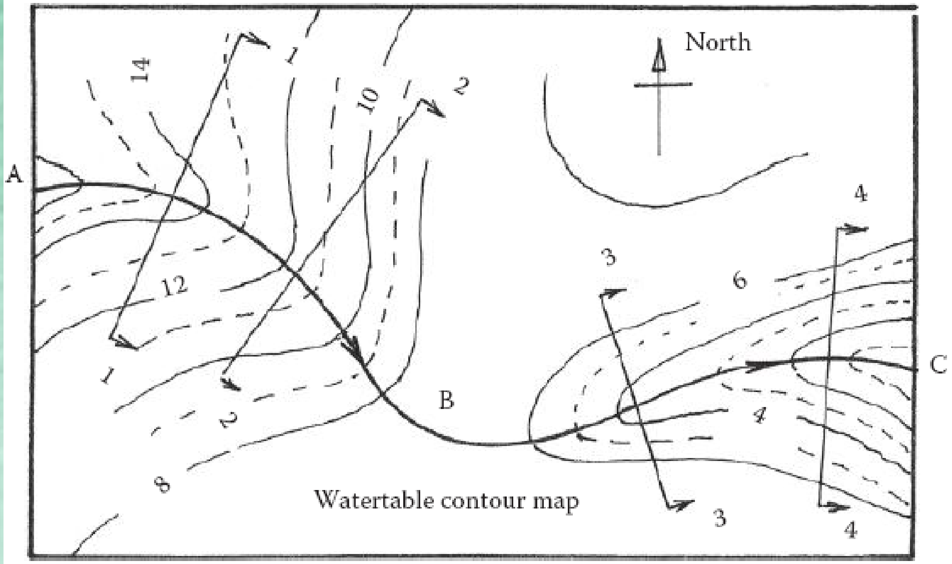


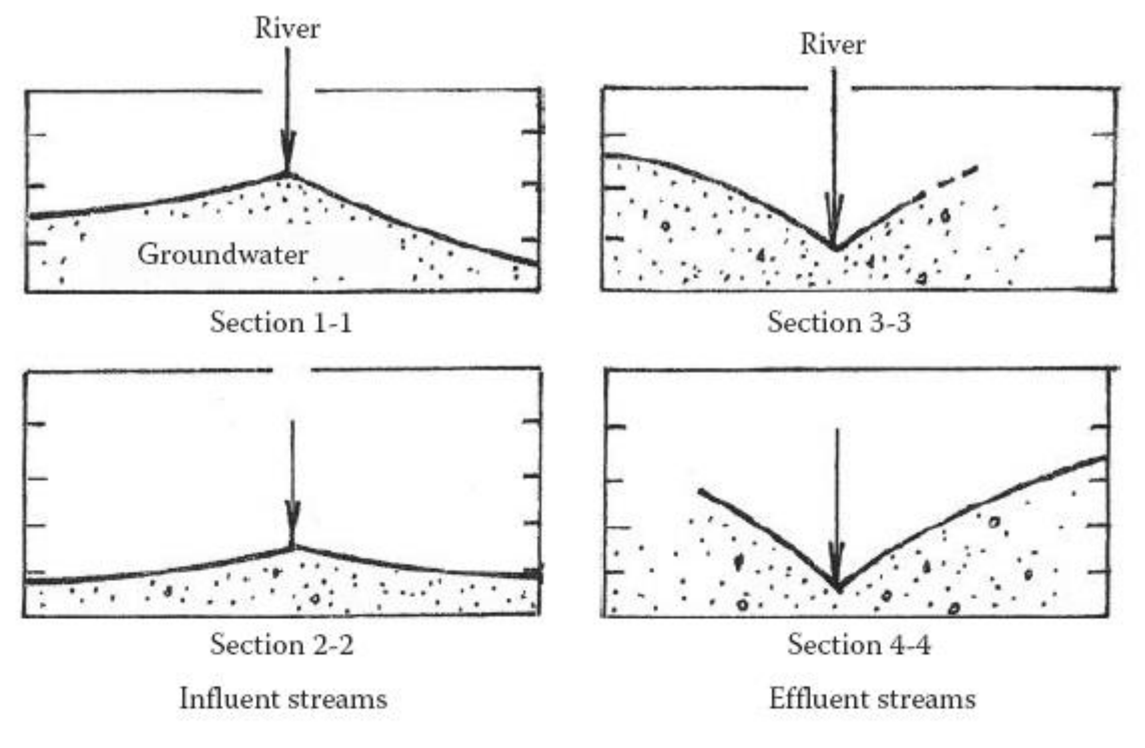
Figure 1.9 River–aquifer interaction: (a) effluent stream, (b) influent stream, (c) influent stream (deep watertable), and (d) bank storage.

مثال

Groundwater contours are shown on the following map along with the course of a river. Determine the river reaches along which the stream behaves like an *effluent* and *influent* stream. Use the data that have been provided on the map for your analysis.



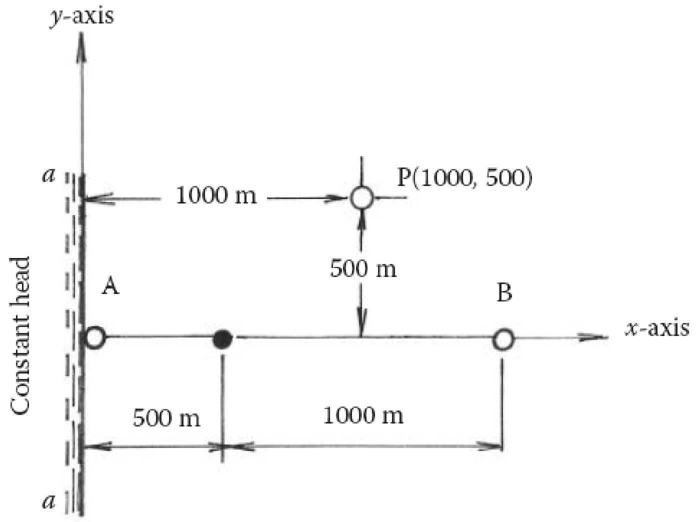
We construct typical cross sections of the phreatic surface across the river. Locations of these sections are identified on the map, and the cross sections themselves are shown in the following drawings. Cross sections, section 1-1 and section 2-2, show that the phreatic surface dips away from the river to the aquifer. Thus, along the reach from section 1-1 to section 2-2, the river behaves like an influent (loosing) stream. Likewise, sections 3-3 and 4-4 show that the phreatic surface slopes toward the river, indicating seepage from the aquifer into the river. Thus, along the reach between these two sections, the river behaves as an effluent (gaining) stream. It may be further observed that influent streams flow over the ridge of the groundwater surface, while the effluent streams flow along the valley of groundwater surface. Thus, a study of the contour map shows that the river reach from A to B and from B to C behaves, respectively, as the influent and effluent stream.

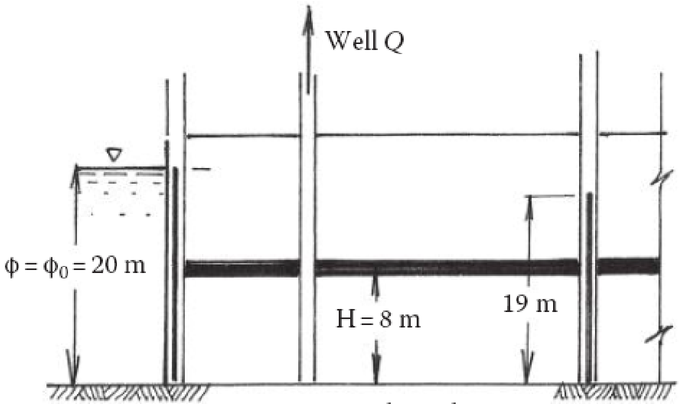

River
Groundwater
Section 1-1
River
Section 3-3
Section 2-2
Section 4-4
Influent streams
Effluent streams

ILLUSTRATIVE PROBLEMS

A well is located at 500 m from a long, straight constant-head boundary *aa*. The aquifer is uniformly isotropic with coefficient of permeability $K = 0.00014$ m/s. The steady-state piezometric levels at observation wells, A and B, are 20 m and 19 m, respectively, as shown in the following sketch. If the aquifer remains confined of uniform depth $H = 8$ m, find the following:

(a) The piezometric level in observation well located at P(1000, 500)
(b) The steady (not changing with time) discharge Q





Part (*a*): This is a problem where the well is situated near a long constant-head boundary. The potential Φ is in general found from the following equation:

$$\Phi = \frac{Q}{2\pi}\ln\left[\frac{r_1}{r_2}\right] + \Phi_0 \quad \text{(IP5.1.1)}$$

Since the aquifer remains confined, the preceding equation can be replaced by the following equation:

$$\phi = \frac{Q}{2\pi KH}\ln\left[\frac{r_1}{r_2}\right] + \phi_0 \quad \text{(IP5.1.2)}$$

where

- ϕ represents the piezometric level at the movable point P(x, y)
- r_1 and r_2 represent the radial distances from the well and its image to the point P(x, y), respectively
- ϕ_0 represents the piezometric level in the observation well A (or the level in the perennial river)

At observation well B, the following information is known:

$r_1 = 1000$ m

$r_2 = 2000$ m

$\phi_B = 19$ m

$\phi_0 = 20$ m

Substituting the preceding information in Equation IP5.1.2 yields the following:

$$19 = \frac{Q}{2\pi KH}\ln\left[\frac{1000}{2000}\right] + 20 \quad \text{(IP5.1.3)}$$

From Equation IP5.1.3, the following can be obtained:

$$\frac{Q}{2\pi KH} = \frac{19-20}{\ln(0.5)} = 1.4427 \text{ m} \qquad \text{(IP5.1.4)}$$

At point P(1000, 500), the following data can be easily obtained:

$r_1 = 707.107$ m

$r_2 = 1581.14$ m

$\phi_0 = 20$ m

Thus, substituting the preceding values in Equation IP5.1.2 yields the following:

$$\phi_P = 1.4427 \ln\left[\frac{707.107}{1581.13}\right] + 20 = 18.84 \text{ m}.$$

Part (b): The steady-state discharge can be obtained from Equation IP5.1.4 as follows:

$$Q = 1.4427 \text{ m} \times 2\pi \times K \times H = 1.4427 \text{ m} \times 2\pi \times 0.00014 \text{ m/s} \times 8 \text{ m}$$
$$= 0.010 \text{ m}^3/\text{s}.$$

Example Calculation

The following example applies this form of the hydrologic budget to calculate safe yield for a project in northern Iran (4) (Figure 20.3).

The time period chosen for the water balance was the four-year period of 1969 to 1972, and it was considered representative of mean geohydrologic conditions.

Ground Water Inflow, Q_i. The inflow term was calculated as

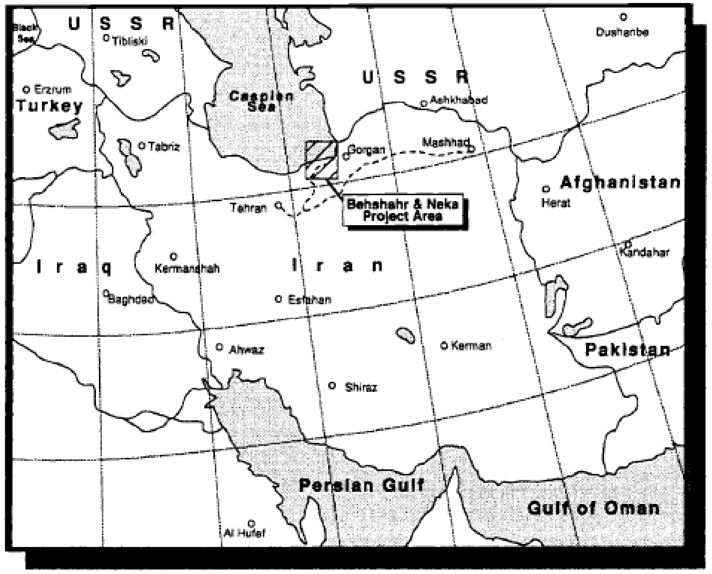
$$Q_i = (L)(Kb)\left(\frac{\Delta h}{\Delta x}\right) = 70{,}542 \text{ acre-ft/year} ,$$

where

Q_i = flow of ground water moving across a section L (normal to the flow path) having an average transmissivity $T(Kb = T)$ under a hydraulic gradient of $\Delta h/\Delta x$.

The recharge area originates at the southern boundary (foothill zone) of the area of investigation. Values of average transmissivity for the area were obtained from aquifer test results correlated with a geophysical resistivity survey. Hydraulic gradients were obtained from water-level contour maps, and cross-sectional areas were drawn parallel to equipotential lines.

Ground Water Outflow Q_0. Ground water outflow across the northern boundary (near the coast of the Caspian Sea) consists of the sum of the outflow from the water table and the deeper confined aquifers. However, as the upper aquifer water-level contour maps show little-to-no outflow, the estimate was based on the deeper aquifers only. The total subsurface ground water outflow across the northern boundary was estimated as 5676 acre-ft/year.



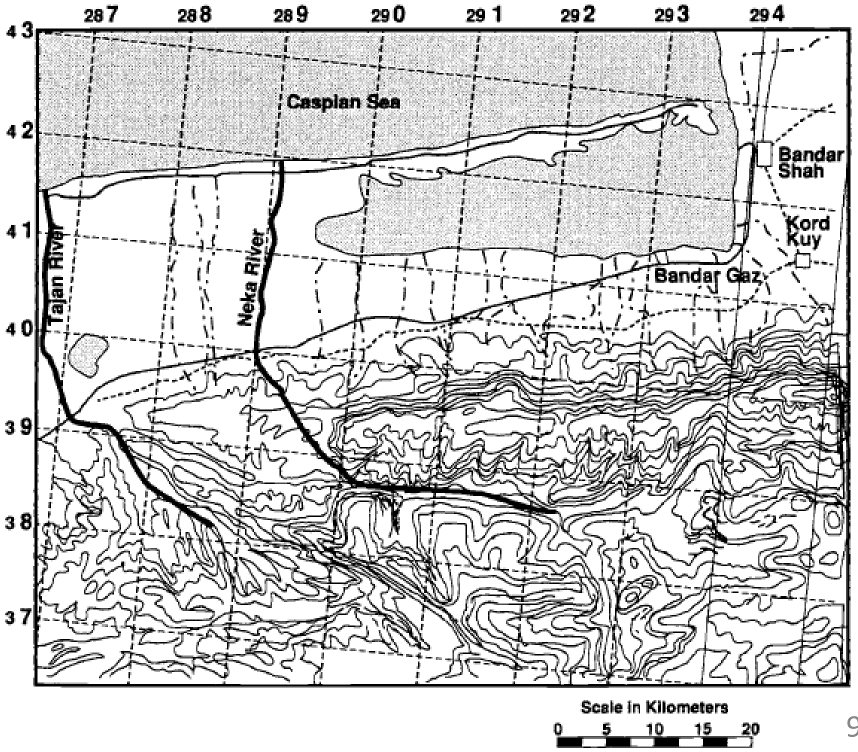


Figure 20.3. Location map of project area.

Conveyance Loss, Q_{cl}. Some water diverted for irrigation is lost during transit and percolates as recharge to the upper aquifer. The conveyance water loss was assumed to be 15% of the total volume of water diverted for irrigation (204,330 acre-ft/yr as measured at the main diversion located at the base of the mountains). Of this, 15% is the conveyance loss, or 30,649 acre-ft/yr.

Effective Precipitation (EP). In areas where the ground water table is reached by plant roots or is so shallow that the capillary fringe extends to the land surface, the amount of water discharged by transpiration through plants and evaporation from the land surface is the "potential evapotranspiration" of that area (5). The effective precipitation or infiltration is that portion of the precipitation that actually

TABLE 20.1 Potential Infiltration and Evapotranspiration Constants

Depth to Shallow Ground Water (ft)	Potential Evapotranspiration Constant, α	Potential Infiltration Constant, β
<3	0.4	0.1
3–15	0.1	0.5
>15	0	0.8

Note: α and β empirically determined for the area (14).

reaches the phreatic water surface. Its calculation involves subtraction of potential evapotranspiration (PE) from potential infiltration (PI). Potential infiltration is estimated by adjusting residual precipitation for soil moisture deficiency and depth to the water table.

In the study area monthly precipitation records were available at two stations. After adjusting for soil moisture deficiency and interception, the average value of residual precipitation was 8.67 in./yr.

Potential Infiltration was then calculated from

$$\text{PI} = P\beta A = 113{,}679 \text{ acre-ft/yr}$$

where

- PI = potential infiltration [acre-ft/yr],
- P = residual precipitation [ft/yr], 0.723 ft/yr,
- β = empirical infiltration factor (function of depth to ground water; see Table 20.1),
- A = total area, 338,567 acres.

Potential evapotranspiration was calculated in a similar manner:

$$\text{PE} = 3.445^{1}\alpha A = 178{,}221 \text{ acre-ft/yr} ,$$

where

- PE = potential evapotranspiration [acre-ft],
- α = empirical evaporation constant (function of depth to water table, see Table 20.1).

Effective precipitation (EP) is the difference between potential infiltration (PI) and potential evapotranspiration (PE):

$$\text{EP} = \text{PI} - \text{PE} = -64{,}542 \text{ acre-ft/yr} .$$

The negative sign signifies a loss from the ground water reservoir.

1. The average of pan evaporation values in the area is 3.445 ft/yr.

Irrigation Return Flow, Q_{rfs} and Q_{rfg}. Of the total water delivered for irrigation in the project area, the following disposition of use was assumed:

1. Consumptive use by plants (60% of total).
2. Evaporation of excess water (15%).
3. Percolation to the phreatic aquifer (25%).

Of the 204,330 acre-ft/yr of surface water diverted for irrigation, 173,680 acre-ft/yr reaches the fields (15% was lost during conveyance). Of this amount 25% or 43,420 acre-ft/yr infiltrates as irrigation return flow recharging the phreatic aquifer (Q_{rfs}).

A total amount of 103,786 acre-ft/yr was pumped from shallow and deep aquifers. Of this, 25% returns to the phreatic aquifer contributing 25,947 acre-ft/yr toward ground water recharge (Q_{rfg}).

Pumping, Q_{lp}. Total irrigation pumping was estimated from averages of the metered yearly discharge for all wells and ghanats in the area. Specifically,

- All shallow and deep wells and ghanats were located on 1:20,000 scale maps.
- Average pumping discharges were recorded from records or personal communication with the landowners in the field.

Average yearly discharge was obtained by weighting pumping days to the total days per year:

$$\overline{Q} = Q \times \frac{t}{365},$$

where

$\overline{Q}$ = average yearly discharge = 103,786 acre-ft/yr,
Q = metered or estimated discharge [acre-ft/yr],
t = days of pumping per year.

Ground Water Storage Change, ΔV. In the project area the ground water reservoir was approximated by a two-layered system:

1. Phreatic (less than 150-ft depth).
2. Deep (sum of all confined and semiconfined aquifers below 150 ft).

As a first approximation, it was assumed that:

$$\Delta V = \delta V_1 + \delta V_2,$$

where

ΔV = total ground water storage change [acre-ft/yr],
δV_1 = phreatic aquifer change [acre-ft/yr],
δV_2 = deep aquifer change [acre-ft/yr].

It was further assumed that $\delta V_2 \rightarrow 0$ (deep aquifer storage). This was a reasonable assumption as total deep aquifer exploitation during the study period was only 44,859 acre-ft/yr, whereas total ground water inflow alone was 70,542 acre-ft/yr.

Shallow aquifer storage change was computed from a plot of average ground water fluctuations (hydrographs) after subdividing the area into polygons (using Thiessen's method) with the shallow wells as the center of each polygon.

The average head (H) for each polygonal area for the time period in question was computed from

$$H = \frac{\sum_{i=1}^{n} A_i h_i}{\sum_{i=1}^{n} A_i}$$

where

A_i = polygonal area [acres],
h_i = water level elevation as measured from the individual well hydrograph [ft],
n = total number of wells measured (74).

The southern recharge area (Foothill Zone) was analyzed separately in order to compare its fluctuations with the fluctuations of the total area of the plain.

The phreatic aquifer storage change was computed from the following equation:

$$\Delta V = A\theta\delta ,$$

where

A = total aquifer surface area [acres],
θ = effective porosity,
δ = slope of hydrographs [ft/yr].

Foothill Zone

$$\Delta V = 87{,}237 \times 0.06 \times 0.0116 = 61 \text{ acre-ft/yr} .$$

Total Plain Area

$$\Delta V = 308{,}912 \times 0.04 \times (-0.00241) = -30 \text{ acre-ft/yr} .$$

As can be see in the calculation, the low slopes indicated that in an average year no significant phreatic aquifer storage change occurred, and so ΔV was assumed to be zero and within the error of estimation of the regional water balance.

Unknown Recharge, Q_{ur}. The error term in the analysis is the sum of all hydrologic components not accounted for in Equation 20.2. The major portion probably consists of natural recharge components neglected in the subsurface inflow/outflow calculations.

Because unknown recharge is the most intangible item in the equation, a balance was forced at its expense. The resulting value of 3477 acre-ft/yr indicates a slight gain in recharge.

4. Ground water outflow from the basin is assumed to be zero.

The resulting annual safe yield (SY) was calculated as

$$\begin{aligned} SY &= Q_i + Q_{cl} + Q_{rfs} + Q_{rfg} + Q_{ur} + EP \\ &= 70{,}542 + 30{,}649 + 43{,}420 + 25{,}947 \\ &\quad + 3477 + 113{,}679 \\ &= 287{,}714 \text{ acre-ft/yr} . \end{aligned}$$

Figure 20.4 summarizes components in the hydrologic equilibrium equation example.

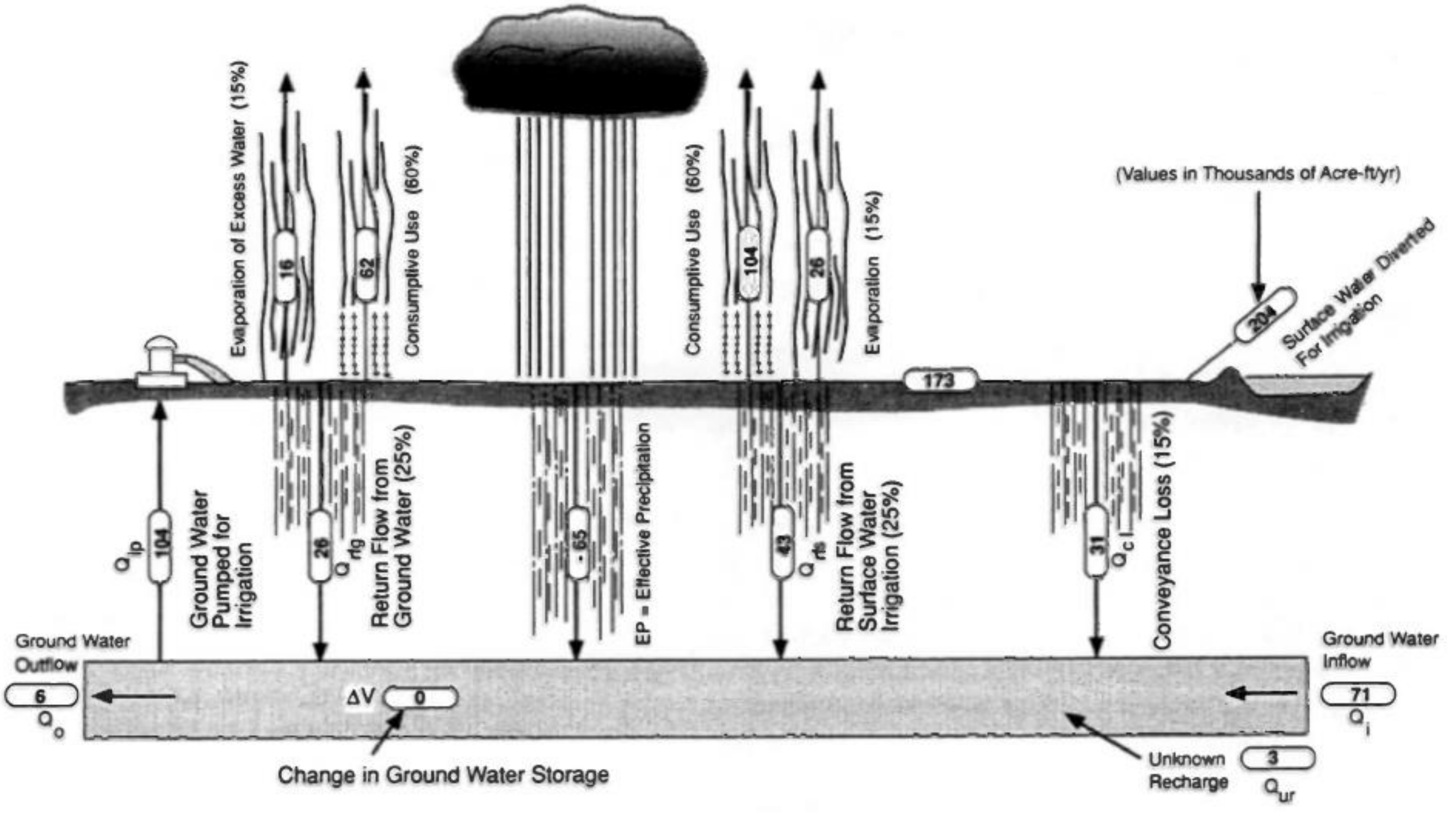
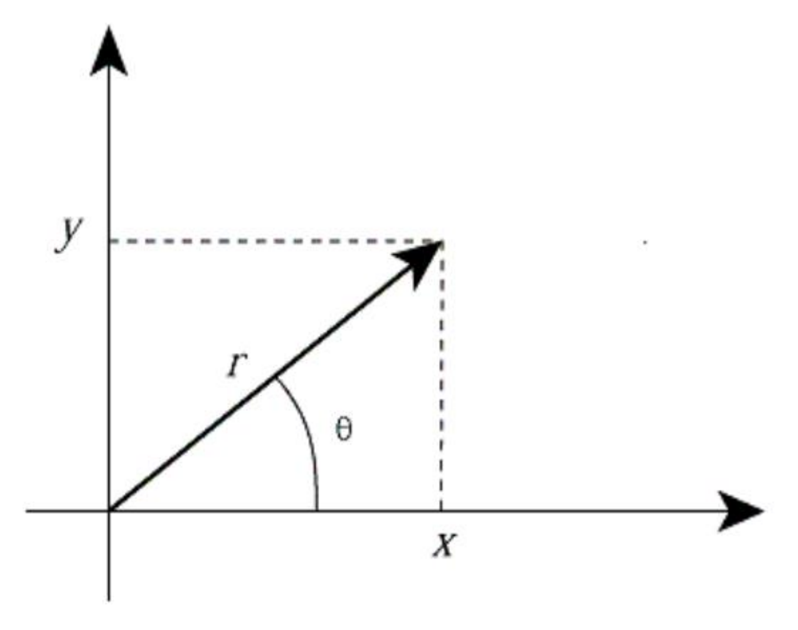


Figure 20.4. Hydrologic equilibrium equation components.

جریان شعاعی غیردائمی



$$x = r\cos\theta$$
$$y = r\sin\theta$$
$$r = \sqrt{x^{۲} + y^{۲}}$$
$$\theta = \tan^{-۱}\frac{y}{x}$$

$$\frac{\partial h^{۲}}{\partial r^{۲}} + \frac{۱}{r}\frac{\partial h}{\partial r} = \frac{S}{T}\frac{\partial h}{\partial t}$$

$$\frac{\partial s^{۲}}{\partial r^{۲}} + \frac{۱}{r}\frac{\partial s}{\partial r} = \frac{S}{T}\frac{\partial s}{\partial t}$$

$$\alpha\left(\frac{\partial^2 s}{\partial r^2} + \frac{1}{r}\frac{\partial s}{\partial r}\right) = \frac{\partial s}{\partial t}$$

$$\begin{cases} in\ confined\ aquifer \quad \alpha = \dfrac{T}{S} \\ in\ unconfined\ aquifer \alpha = \dfrac{T}{S_{ya}} \end{cases}$$

✓لایهٔ آبدار همگن و همروند بوده و گسترش آن بی نهایت است.
✓قابلیت انتقال لایهٔ آبدار T ثابت است.
✓مقدار آبی که پمپاژ می شود از ذخیرهٔ لایهٔ آبدار استخراج می شود.
✓عمق حفاری چاه کل لایهٔ آبدار را در بر گرفته و قطر چاه بسیار کوچک است.

$$s = \frac{Q}{۴\pi T}\int_{u}^{\infty}\frac{e^{-u}}{u}du$$

این رابطه به نام تیس معروف است و بخش انتگرال آن را تابع چاه گویند و با علامت W(u) نشان می‌دهند.
بنابراین خواهیم داشت

$$W(u) = \int_{u}^{\infty}\frac{e^{-u}}{u}du$$

$$s = \frac{Q}{۴\pi T}W(u)$$

$$u = \frac{r^{۲}S}{۴Tt}$$

$$s = \frac{Q}{۴\pi T}\left[-۰/۵۷۷۲ - \ln u + u - \frac{u^{۲}}{۲\times ۲!} + \frac{u^{۳}}{۳\times ۳!} - \frac{u^{۴}}{۴\times ۴!} + \cdots\right]$$

در این روابط: r فاصله نقطه مورد نظر تا چاه پمپاژ؛ t زمان شروع از پمپاژ؛ s افت سطح آب در نقطه مورد نظر؛ S ضریب ذخیره و T قابلیت انتقال است.

مقادیر W(u) به ازای مقادیر مختلف u

u	۱/۰	۲/۰	۳/۰	۴/۰	۵/۰	۶/۰	۷/۰	۸/۰	۹/۰
×۱	۰/۲۱۹	۰/۰۴۹	۰/۰۱۳	۰/۰۰۳۸	۰/۰۰۱۱	۰/۰۰۰۳۶	۰/۰۰۰۱۲	۰/۰۰۰۰۳۸	۰/۰۰۰۰۱۲
$\times ۱۰^{-۱}$	۱/۸۲	۱/۲۲	۰/۹۱	۰/۷۰	۰/۵۶	۰/۴۵	۰/۳۷	۰/۳۱	۰/۲۶
$\times ۱۰^{-۲}$	۴/۰۴	۳/۳۵	۲/۹۶	۲/۶۸	۲/۴۷	۲/۳۰	۲/۱۵	۲/۰۳	۱/۹۲
$\times ۱۰^{-۳}$	۶/۳۳	۵/۶۴	۵/۲۳	۴/۹۵	۴/۷۳	۴/۵۴	۴/۳۹	۴/۲۶	۴/۱۴
$\times ۱۰^{-۴}$	۸/۶۳	۷/۹۴	۷/۵۳	۷/۲۵	۷/۰۲	۶/۸۴	۶/۶۹	۶/۵۵	۶/۴۴
$\times ۱۰^{-۵}$	۱۰/۹۴	۱۰/۲۴	۹/۸۴	۹/۵۵	۹/۳۳	۹/۱۴	۸/۹۹	۸/۸۶	۸/۷۴
$\times ۱۰^{-۶}$	۱۳/۲۴	۱۲/۵۵	۱۲/۱۴	۱۱/۸۵	۱۱/۶۳	۱۱/۴۵	۱۱/۲۹	۱۱/۱۶	۱۱/۰۴
$\times ۱۰^{-۷}$	۱۵/۵۴	۱۴/۸۵	۱۴/۴۴	۱۴/۱۵	۱۳/۹۳	۱۳/۷۵	۱۳/۶	۱۳/۴۶	۱۳/۳۴
$\times ۱۰^{-۸}$	۱۷/۸۴	۱۷/۱۵	۱۶/۷۴	۱۶/۴۶	۱۶/۲۳	۱۶/۰۵	۱۵/۹	۱۵/۷۶	۱۵/۶۵
$\times ۱۰^{-۹}$	۲۰/۱۵	۱۹/۴۵	۱۹/۰۵	۱۸/۷۶	۱۸/۵۴	۱۸/۳۵	۱۸/۲	۱۸/۰۷	۱۷/۹۵
$\times ۱۰^{-۱۰}$	۲۲/۴۵	۲۱/۷۶	۲۱/۳۵	۲۱/۰۶	۲۰/۸۴	۲۰/۶۶	۲۰/۵	۲۰/۳۷	۲۰/۲۵
$\times ۱۰^{-۱۱}$	۲۴/۷۵	۲۴/۰۶	۲۳/۶۵	۲۳/۳۶	۲۳/۱۴	۲۲/۹۶	۲۲/۸۱	۲۲/۶۷	۲۲/۵۵
$\times ۱۰^{-۱۲}$	۲۷/۰۵	۲۶/۳۶	۲۵/۹۶	۲۵/۶۷	۲۵/۴۴	۲۵/۲۶	۲۵/۱۱	۲۴/۹۷	۲۴/۸۶
$\times ۱۰^{-۱۳}$	۲۹/۳۶	۲۸/۶۶	۲۸/۲۶	۲۷/۹۷	۲۷/۷۵	۲۷/۵۶	۲۷/۴۱	۲۷/۲۸	۲۷/۱۶
$\times ۱۰^{-۱۴}$	۳۱/۶۶	۳۰/۹۷	۳۰/۵۶	۳۰/۲۷	۳۰/۰۵	۲۹/۸۷	۲۹/۷۱	۲۹/۵۸	۲۹/۴۶
$\times ۱۰^{-۱۵}$	۳۳/۹۶	۳۳/۲۷	۳۲/۸۶	۳۲/۵۸	۳۲/۳۵	۳۲/۱۷	۳۲/۰۲	۳۱/۸۸	۳۱/۷۶

ژاکوب نیز راه حل ساده تری را برای معادلهٔ تیس ارائه نمود. اگر مقدار u کوچک باشد (مثلاً کمتر از ۰/۰۱)

$$s = \frac{Q}{۴\pi T}\left[-۰/۵۷۷۲ - \ln\left(\frac{r^۲ S}{۴Tt}\right)\right]$$

می توان مستقیماً مقدار s (افت سطح پیزومتریک) را بر حسب r و t برای مقادیر معلوم Q و T و S حساب کرد

$$s = \frac{Q}{۴\pi T}\left(\ln\frac{۲/۲۵T \times t}{r^۲ S}\right) \text{ یا } s = \frac{۲/۳Q}{۴\pi T}\left(\log\frac{۲/۲۵T \times t}{r^۲ S}\right)$$

در مورد لایهٔ آبدار آزاد در جریان غیر ماندگار چون در حین پمپاژ با پائین رفتن سطح ایستابی، مقدار T ($T=K\times D$) همراه با t و r تغییر می کند و مؤلفه جریان قائم هم ممکن است در نزدیک چاه زیاد شود، بنابراین مسئله خیلی پیچیده تر می شود. ولی اگر مقدار افت در مقایسهٔ با ضخامت لایه آبدار کم باشد، می توان از راه حل های تیس و ژاکوب استفاده کرد